MADE EASY

Civil Engineering

Toppers Handwritten Notes

ENGINEERING MACHANICS

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* Engineering Mechanics

→ "It is a science which deals and predicts the condition of the system either at rest or in motion under the action of external force."

Engineering Mechanics

- Rigid Bodies
  - Statics
    - → Different type of forces
    - → Vectors
    - → Newtons I & III law (NFL) (NTL)
    - → Equilibrium
    - → Different type of Equlibrium.
    - → Torque
    - → Truss
    - → work
    - → Principle of Virtual work.
    - → Centre of mass and centroid.
  - Dynamics.
    - Kinematics
    - Kinetics
    - • Newtons II law of motion (NSL)
    - • Different type of motion.
    - • Rectilinear translation
    - • Circular motion
    - • friction
    - • Rotation
    - • General motion.
- Deformable bodies
  - Solids
    - Statics
    - Dynamics
      - Kinematics
      - Kinetics
  - Fluids
    - Statics
    - Dynamics
      - Kinematics
      - Kinetics

Different ideal concepts in engineering mechanics

1> Rigid body

→ whenever loads applied on body, body deforms but if the deformations are negligible wrt size of the body then we can neglect those deformations and we can treat the bodies as a rigid body.

2> Continuum

→ Even in solids there is void space between the adjucent molecules and atoms we know that these void spaces are microscopic therefor if the size of body is sufficiently good that means microscopic then we can neglect the void spaces and we can assume adjucent to one molecule there is another molecule hence the entire body is treated as continuous distribution of mass known as continuum.

3> $\underbrace{\text{Body}}_{\text{Real}}$ as a $\underbrace{\text{Particles}}_{\text{Real}}$

Force ($\vec{F}$)

→ Action of one body to the other body.

Vector Quantity

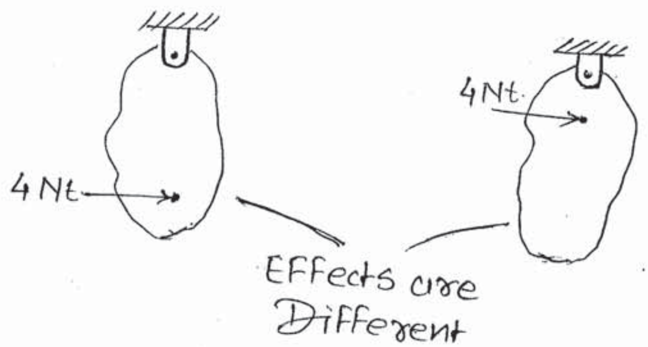
→ Quantities having magnitude and direction.

- when the force is applied on the body this implies that it is applied on some of the particles of body.

Then to define force:

- Magnitude
- Direction
- Point of application

} Required.



whenever the force is applied on the body, then for that force ($\vec{F}$), two bodies will exist.

- → One body → which is applying force
- → Second body → on which the force is applied.

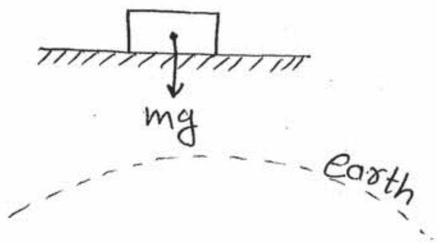
Note

→ If a force is acting on the body, but there is no other body which is applying this force, that force is called Pseudo force (Artificial force)

Different type of forces
[most frequently appearing in EM]

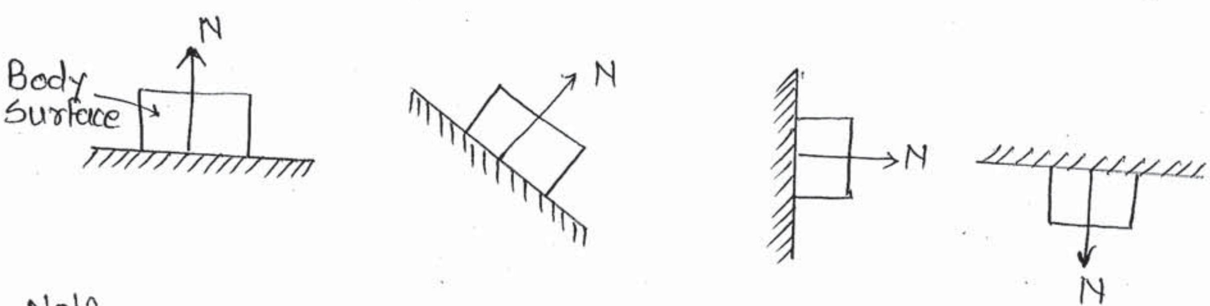
1) **Weight (ω) (mg)**

→ force acted on the body by the earth.
→ It is a body force.



2) **Normal Reaction (N) :-**

→ Surface force
→ Acts on the body by the surface exactly in the direction perpendicular to the surface.
→ It is due to pressing effect between contacting surface.



Note

→ IF the surface are touching but not pressing then,

$\boxed{N = 0}$ **

3) **Friction: (Dry friction)**

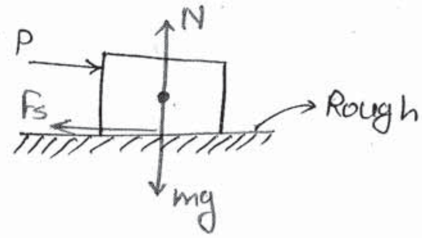
→ Surface force
→ Along the surface
→ It resists the relative motion or tendency or relative motion between the contacting surface.

Static friction (F_s)

→ Due to the tendency of relative motion between the contacting surface {no relative motion}.

→ It is a variable friction.

$\boxed{0 \leq F_s \leq \mu_s N}$ $\mu_s \to$ Coefficient of static friction.



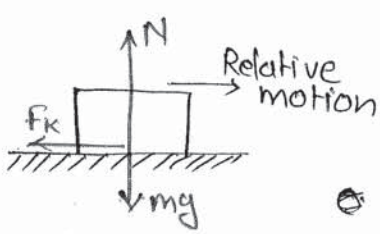
Applied force	Static friction (f_s)
0	0
1 Nt	1 Nt
2 Nt	2 Nt
3 Nt	3 Nt
⋮	⋮
$\mu_s N$	$\mu_s N$

→ Static friction is conservative force

$\boxed{\text{Energy loss} = 0}$ **

It is a tendency of relative motion is more than the $F_{s\,max} = \mu_s \cdot N$.

- If relative motion starts friction developed is called kinematic friction (F_K) ~~due~~ this friction is developed due to the relative motion between the contacting surfaces.



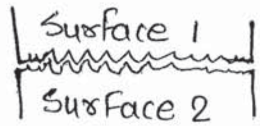
$\boxed{F_K = \mu_K \cdot N}$

$\mu_K \to$ Coeff. of Kinetic friction

Constant friction = Non ~~[illegible]~~ conservative force
Energy loss.

Coefficient of Friction (μ_s, μ_k)

→ Every surface is having surface irregularities

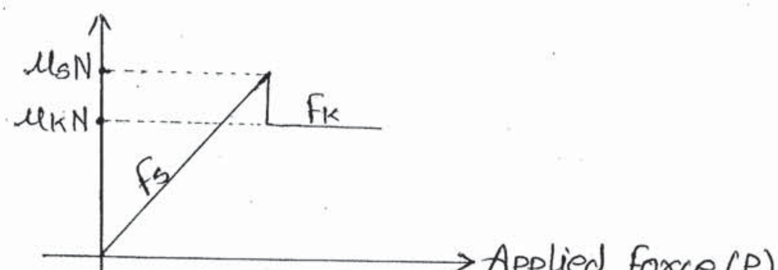


Depends upon

1) Surface irregularities
2) How irregularities are interlocked.
3) No. of interlocking.

"μ_s" is slightly more than "μ_k"

→ Because a little bit decrease in strength of interlocking at the moment when relative motion starts.

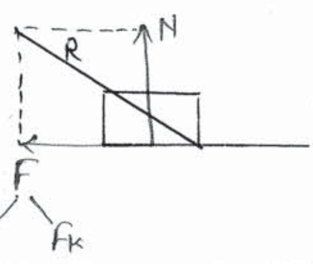


Total Contact Force: ($\vec{R}$)

$$\boxed{\vec{R} = \vec{N} + \vec{F}}$$

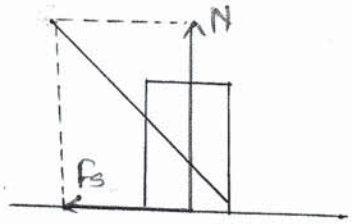
$\vec{F}$: f_s, f_k.

Resultant of friction & normal reaction.



Angle of static friction (ϕ_s)

→ Angle between the normal reaction and total contact forces when body is at verge of Relative motion.

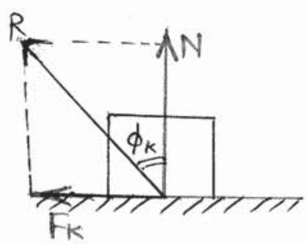


$$\frac{\cancel{R}\sin\phi_s = f_{s\,max} = \mu_s \cancel{N}}{\cancel{R}\cos\phi_s = \cancel{N}}$$

$$\boxed{\mu_s = \tan\phi_s}\ **$$

Angle of kinetic friction (ϕ_K)

→ Angle between normal reaction and total contact force when body is in relative motion.



$$\frac{\cancel{R}\sin\phi_K = f_K = \mu_K \cancel{N}}{\cancel{R}\cos\phi_K = \cancel{N}}$$

$$\boxed{\mu_K = \tan\phi_K}\ **$$

Note • If only one coefficient of friction. (μ)

$\Rightarrow \boxed{\mu_s = \mu_K = \mu}$

• If only one angle of friction (ϕ) is given.

$$\boxed{\mu_s = \mu_K = \tan\phi_s = \tan\phi_K = \tan\phi = \mu}$$

—x—

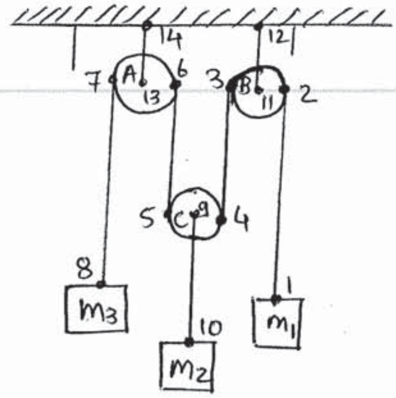
4) Tension (Tension in string):-

→ It is a pulling force.

→ Tension always acts along the string.

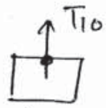
→ It is always away from the body (system).

Consider the following system.

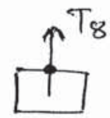


$\underline{m_1}$:

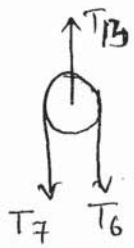
$\underline{m_2}$:



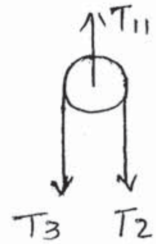
$\underline{m_3}$:



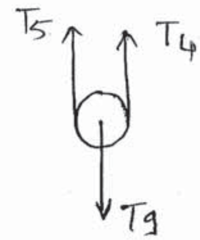
Pulley A:



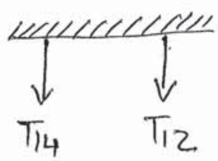
Pulley B:



Pulley C:



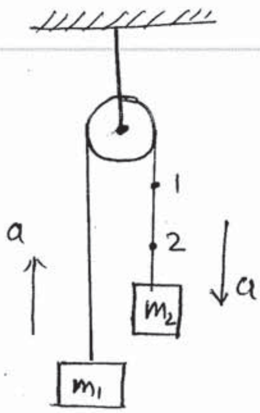
Support

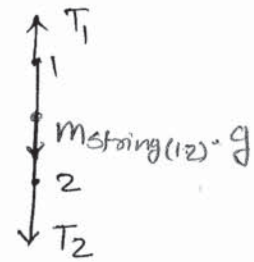


1-2 Portion of string

- Variation in Tension along the length of string :-

String (1-2) Portion [F.B.D]





By NSL

$$T_2 + m_{string(12)} \cdot g - T_1 = m_{string(12)} \cdot a \quad \text{—— (1)}$$

If string is massless

$$m_{string(12)} = 0$$

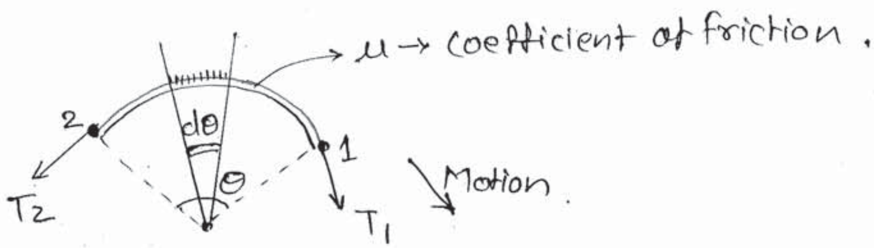
eq^n (1) $\Rightarrow$ $T_2 - T_1 = 0$

$$\boxed{T_2 = T_1} \; **$$

ie; Tension along the string will remain same at every point.

- Variation in Tension of string which is wrapped over pulley (Surface)

(string is massless).



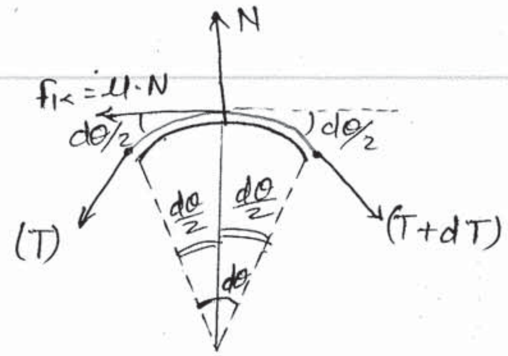
$\theta \rightarrow$ Angle of wrap.

$$T_1 > T_2$$

$T_1 \rightarrow$ Tight side Tension

$T_2 \rightarrow$ Slag side Tension

→ Differential element of string (FBD)



$$N = T \cdot \sin\frac{d\theta}{2} + (T+dT)\sin\frac{d\theta}{2} = T \cdot \frac{d\theta}{2} + (T+dT)\frac{d\theta}{2}$$

$$N = T \cdot \frac{d\theta}{2} + T \cdot \frac{d\theta}{2} + \cancel{\frac{dT \cdot d\theta}{2}}^{\to 0} = \underline{T \cdot d\theta}$$

now,

$$(T+dT)\cos\frac{d\theta}{2} - T \cdot \cos\frac{d\theta}{2} - \mu N = (\cancel{m_{string}})^{\to 0} \cdot a$$

$$(\cancel{T}+dT) - \cancel{T} - \mu \cdot T \cdot d\theta = 0$$

$$dT = \mu \cdot T \cdot d\theta$$

$$\int_{T_2}^{T_1} \frac{dT}{T} = \mu \int_0^{\theta} d\theta$$

$$\left[\ln T\right]_{T_2}^{T_1} = \mu\theta$$

$$\ln\frac{T_1}{T_2} = \mu\theta \implies \frac{T_1}{T_2} = e^{\mu\theta}$$

$$\boxed{T_1 = T_2 e^{\mu\theta}} \;**$$

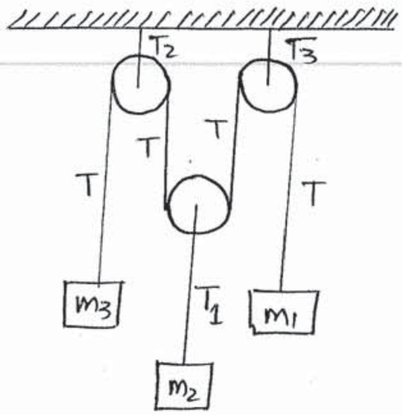
$$T_1 > T_2$$

<u>IF Pulley is friction less</u>

$$\mu = 0$$

$$\Rightarrow \boxed{T_1 = T_2} \;***$$

⇒ If string is massless and pulley is frictionless.



5) Spring force (F_{spring})

→ It is Pulling force or Pushing force.

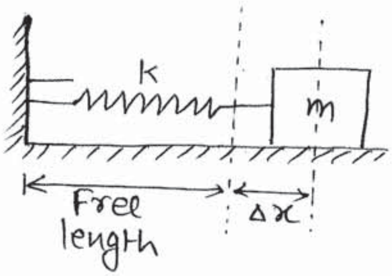
$$\boxed{F_{spring} = k \cdot \Delta x} \; *$$

k → stiffness or spring constant.

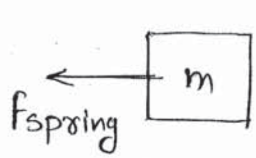
Δx → change in length wrt it's free (Natural) length.

→ If Δx → Elongation

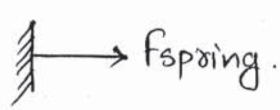
Spring ~~length~~ Force → Pulling



m:

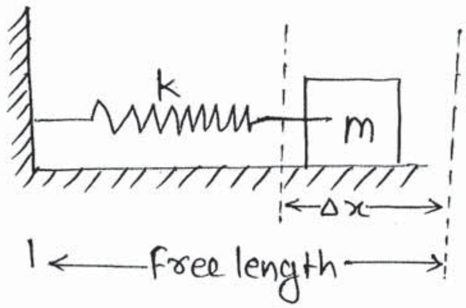


Support

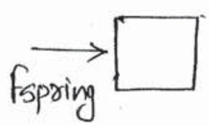


→ If Δx → compressive

Spring force → Pushing

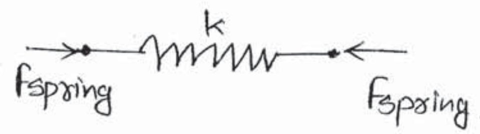


m:



Support

Spring

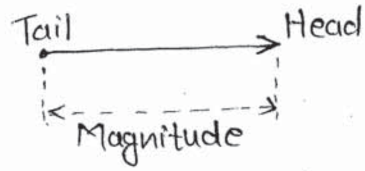


Vectors

→ Quantities having magnitude and direction are called Vectors.

- Graphical Representation

↓ Arrow



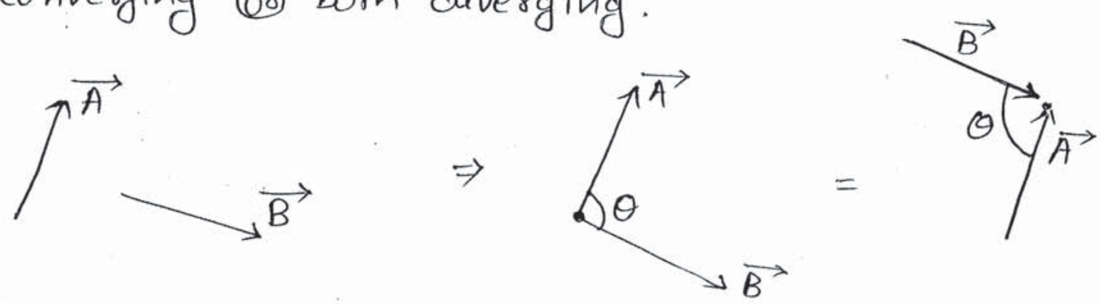
Direction :- From Tail to head.

Note

Any vector can be shifted at any place provided it's magnitude and direction remains same.

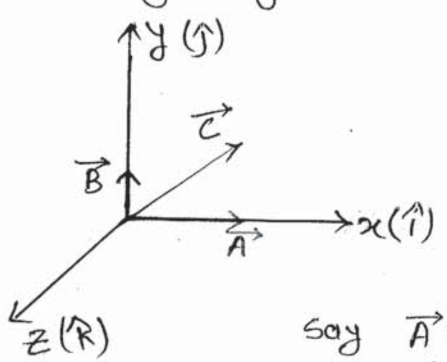
- Angle b/w the vectors (θ)

→ Minimum angle b/w the vectors at a point, either both converging (or) both diverging.



Unit Vectors ($\hat{\ }$)

→ Having magnitude Unity and are used to represent the direction.



$\hat{i}$ → Unit vector in x direction

$\hat{j}$ → —— " —— Y —— " ——

$\hat{k}$ → —— " —— Z —— " ——

Say $\vec{A} = 5\,\hat{i}$

$\vec{B} = 2\,\hat{j}$

$\vec{C} = 7\hat{i} + 3\hat{j}$

$$\vec{C} = 7\hat{i} + 3\hat{j}$$

Magnitude,

$$C = \sqrt{7^2 + 3^2} = \sqrt{58}$$

$$\vec{C} = C\hat{C}$$

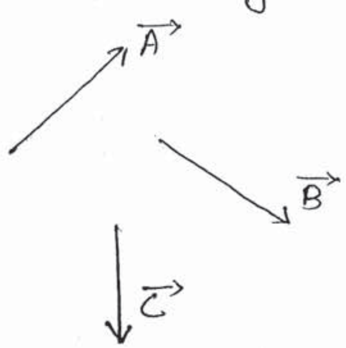
$$\boxed{\hat{C} = \frac{\vec{C}}{C} = \frac{1}{\sqrt{58}}(7\hat{i} + 3\hat{j})}$$

Triangle law of Vector Addition

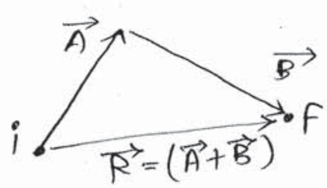
→ According to this law, "in addition of the vectors the head of one vector is joint with the tail ~~of~~ other vector."

The ~~net~~ addition ~~of~~ the vectors is also known as resultant vector.

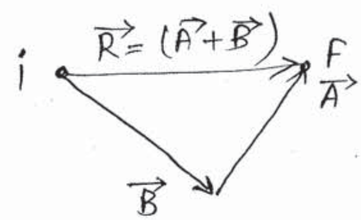
consider following examples,



i) $\vec{A} + \vec{B}$:



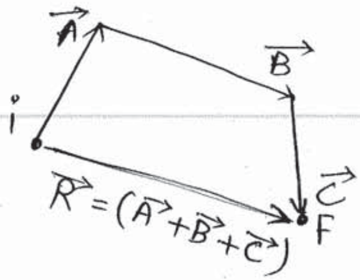
or



i → initial point

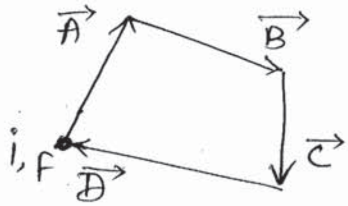
f → final point.

ii) $\vec{A} + \vec{B} + \vec{C}$:



Note

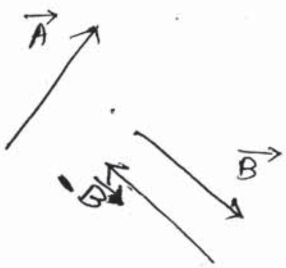
$\boxed{\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0} \Rightarrow$ Starting & Ending point are same.

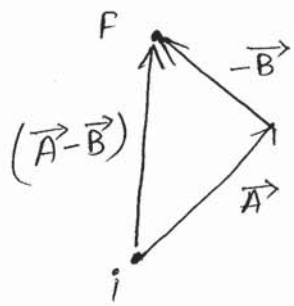


Note

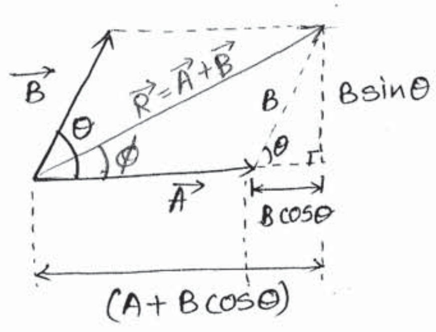
$\vec{A} - \vec{B}$:

$\Rightarrow \vec{A} - \vec{B}$
$= \vec{A} + (-\vec{B})$





Parallelogram law of vector Addition



$\theta \rightarrow$ Angle between the vectors $\vec{A}$ & $\vec{B}$.

$$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$
$$= A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

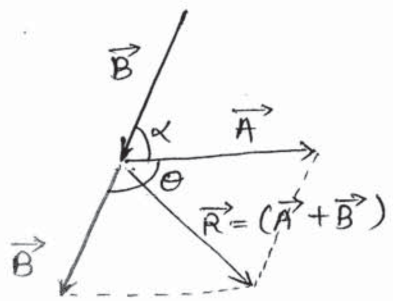
$$R^2 = (A^2 + B^2 + 2AB\cos\theta)$$

$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}} \ *$$

$$\tan\phi = \frac{B\sin\theta}{A+B\cos\theta}$$

$$\boxed{\phi = \tan^{-1}\left\{\frac{B\sin\theta}{A+B\cos\theta}\right\}}\ *$$

<u>Note</u>



$$\theta = (180^\circ - \alpha)$$

$$R = \sqrt{A^2+B^2+2AB\cos(180^\circ-\alpha)}$$

$$\boxed{R = \sqrt{A^2+B^2-2AB\cos\alpha}}$$

Product of Vectors

- → Dot Product (Scalar Product)
- → Cross Product (Vector Product)

i) Dot Product (Scalar Product)

→ It's out come is scalar.

$$\boxed{\vec{A}\cdot\vec{B} = A.B\cos\theta}\ **$$

$\theta \rightarrow$ Angle b/w vector $\vec{A}$ & $\vec{B}$

IF $\theta = 0^\circ$

$$\boxed{\vec{A}\cdot\vec{B} = AB}$$

IF $\theta = 90^\circ$

$$\boxed{\vec{A}\cdot\vec{B} = 0}$$

Note

$\hat{i}\cdot\hat{i} = 1$ | $\hat{i}\cdot\hat{j} = 0$

$\hat{j}\cdot\hat{j} = 1$ | $\hat{j}\cdot\hat{k} = 0$

$\hat{k}\cdot\hat{k} = 1$ | $\hat{k}\cdot\hat{i} = 0$

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\vec{B}\cdot\vec{A} = BA\cos\theta$$

$$\Rightarrow \boxed{\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}}^{**} \longrightarrow$$ Dot Product is commutative.

ii) Cross-Product (Vector-Product)

→ It's ~~ddddd~~ outcome is vector

$$\boxed{\vec{A}\times\vec{B} = (AB\sin\theta)\,\hat{n}}$$

$\theta \rightarrow$ Angle b/w $\vec{A}$ & $\vec{B}$

$\hat{n} \rightarrow$ Unit vector in the direction $\perp^{er}$ to the plane Containing $\vec{A}\times\vec{B}$.

IF $\theta = 0°$

$\vec{A}\times\vec{B} = 0$

IF $\theta = 90°$

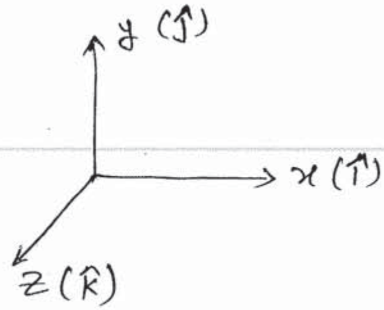
$\vec{A}\times\vec{B} = AB\,\hat{n}$

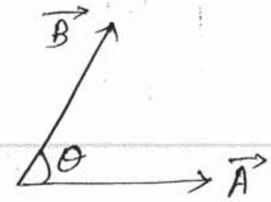
→ Direction of any rotating vector (cross-Product) ($\vec{\omega}$, $\vec{\alpha}$, $\vec{\tau}$ etc) is determined by Right hand thumb rule.

1) Place the right hand in such a way such that thumb is $\perp^{er}$ to the fingers

2) Now rotate the fingers in sense of rotation ($\vec{A}\times\vec{B} \Rightarrow$ Rotate $\vec{A}$ towards $\vec{B}$) then the direction of thumb will indicate the direction of rotating Vector $\vec{A}\times\vec{B}$.

Ex





$$* \boxed{(\vec{A} \times \vec{B}) = (AB\sin\theta)\ \hat{k}} *$$

$$\boxed{\vec{B} \times \vec{A} = (BA\sin\theta)\ (-\hat{k})} *$$

Note

$$\boxed{\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}} **$$

- $\left[\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0\right]$

- $\left[\begin{array}{l} \hat{i} \times \hat{j} = \hat{k} \\ \hat{j} \times \hat{k} = \hat{i} \\ \hat{k} \times \hat{i} = \hat{j} \end{array}\right]$