MADE EASY

Civil Engineering

Toppers Handwritten Notes

FLUID MECHANICS

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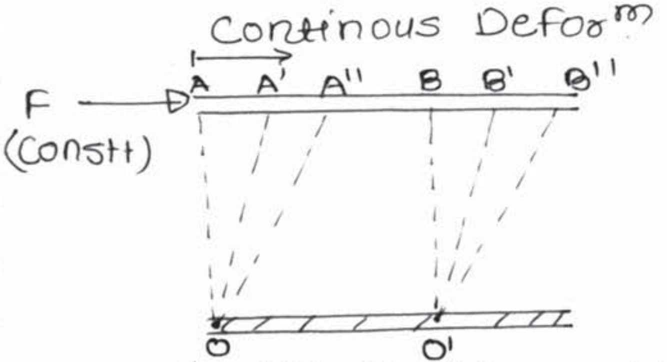
FLUID MECHANICS

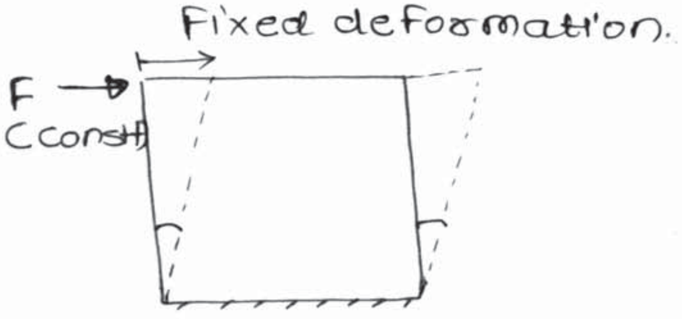
In General

→ Solid

→ Liquid ⎤
→ Gases ⎦ → Fluids



In Fluid the rate of deformation is important



Solid

*Fluid →
Fluid is the substance that deform continously under the application of tangential force, No matter how small it may be

*Fluid has a Continuum → In micro system, When the enter molecular distances are negligible as compare to dimension of system we can assume that adjacent to one molecule, their is another molecular without space theirfore the entire fluid system can be treated as continous distribution of mass and such continous of fluid is known as continuum.

$$\boxed{\text{knudeson No.}(Kn) = \frac{\lambda}{L}}$$

$Kn < 0.01$
"Continnum is valid"

λ = mean free path
L = Characteristic Dimension

Not our Study

- $0.01 < Kn < 0.1$ → Slip Flow
- $0.1 < Kn < 10$ → Transition Flow
- $Kn > 10$ → Free Molecular Flow

→ Fluid property is such that density etc. can be defined as continous function of space variable.

→ Continuum is invalid at very low pressure [At High Elevation]

* Fluid Properties →

① Density (ρ) →

It is defined as mass per unit volume of a substance.

$$\boxed{\rho = \frac{M}{\forall}}$$

$\forall$ = Volume
M = mass

Units → In MKS - kg/m^3
In C.G.S - gm/cm^3

$$\boxed{1\,gm/cm^3 = 1000\,kg/m^3}$$

② Specific Gravity (S) →

$$\boxed{S = \frac{\text{Density of Fluid Substance}}{\text{Density of Standard Fluid Substance}}}$$

For Fluids → Std. Fluid → H_2O at 4°C
$\rho_w = 10^3\,kg/m^3$.

For Gases → Std ~~Gas~~ Fluid ⇒ Air.

Example: $S_{Hg} = 13.6$ {Specific Gravity}

$$\frac{\rho_{Hg}}{\rho_w} = 13.6$$

$$\boxed{\rho_{Hg} = 13.6 \times 10^3 \ kg/m^3}$$

③ Specific weight
or
Weight Density:

It is defind as the weight of the substance per unit volume.

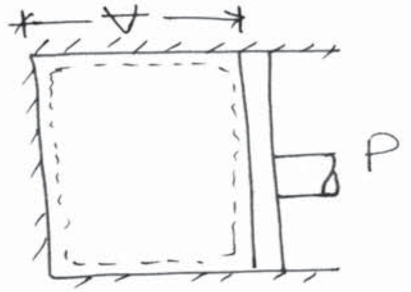
$$\left[\text{Specific weight} = \frac{m \times g}{\forall}\right]$$

$$\boxed{Sp.\ wt. = \rho g}$$

④ Compressibility (β): It is defined as the reciporcal of bulk modulus of elasticity of Fluid.

$$\boxed{\beta = \frac{1}{K}}$$

K = Bulk modulus of elasticity.



$$\left\{K = -\frac{dP}{\frac{d\forall}{\forall}}\right\}$$

$m = \rho \cdot \forall$

$dm = \rho \cdot d\forall + \forall d\rho$

$-\rho d\forall = \forall d\rho$

$$\left\{-\frac{d\forall}{\forall} = \frac{d\rho}{\rho}\right\}$$

$$K = \frac{dP}{\frac{d\rho}{\rho}}$$

$$\boxed{K = \rho \frac{dP}{d\rho}}$$

$$\boxed{\beta = \frac{1}{\rho} \cdot \frac{d\rho}{dP}}$$

→ if density is not changing w.r.t pressure

$$\frac{d\rho}{dP} = 0 \quad , \quad \boxed{\beta = 0} \rightarrow \text{(incompressible)}$$

→ if the density is changing w.r.t pressure

$$\frac{d\rho}{dP} \neq 0 \quad , \quad \boxed{\beta \neq 0} \rightarrow \text{(compressible)}$$

* Liquids

$T = 20°C$

$P = 1\,atm \; , \; \rho_w = 998\ kg/m^3$

$P = 100\,atm \; , \; \rho_w = 1003\ kg/m^3$

$$\%\ \text{change} = \frac{1003 - 998}{998} \times 100$$

$$\simeq 0.5$$

$$\beta \simeq 0 \quad \text{(Incompressible)}$$

* Generally, Liquids are treated as incompressible

* Gases

$$\boxed{P = \rho R T} \rightarrow \text{Highly compressible}$$

Note $\left\{ \text{Mach no (Ma)} = \frac{V}{C} \right\}$ → Velocity of sound in the medium.

{ if $Ma \leq 0.3$, the flow is considered as incompressible.

* Ideal Gas eq^n

① $\boxed{P\cdot\forall = n\cdot\bar{R}\cdot T}$ Absolute temp

Universal Gas Const.

Absolute pressure

No. of moles

② $$P\forall = \frac{m}{M}\cdot\bar{R}\cdot T$$

$$P\forall = m\cdot\frac{\bar{R}}{M}\cdot T$$

$$\boxed{P\cdot\forall = m\cdot R\cdot T}$$

Characteristic Gas Constt.

③ $$P = \frac{m}{\forall}\cdot R\cdot T$$

$$\boxed{P = \rho R T}$$

* Isothermal compressibility OF Gases

$$P\forall = \text{Constt.}$$

$$P\rho^{-1} = \text{constt.}$$

$$\rho^{-1}dP - \rho^{-2}P\cdot d\rho = 0$$

$$\rho^{-1}\left[dP - P\cdot\frac{d\rho}{\rho}\right] = 0$$

$$dP = P\cdot\frac{d\rho}{\rho}$$

$$\frac{dP}{d\rho/\rho} = P$$

$$\boxed{K_{iso} = P}$$

$$\boxed{\beta_{iso} = \frac{1}{P}}$$

* Adiabatic compressibility OF Gases

$$P\cdot\forall^{\gamma} = \text{Constt.}$$

$$P\left(\frac{m}{\rho}\right)^{\gamma} = \text{Constt.}$$

$$P\rho^{-\gamma} = \text{Constt.}$$

$$\rho^{-}dP - \gamma\cdot\rho^{-\gamma-1}P\,d\rho = 0$$

$$\rho^{-\gamma}\left(dP - \gamma\cdot P\frac{d\rho}{\rho}\right) = 0$$

$$dP = \gamma\cdot P\cdot\frac{d\rho}{\rho}$$

For monoatomic Gases $\gamma = 1.67$

$$\frac{dP}{d\rho/\rho} = \gamma\cdot P$$

For diatomic Gases eg: Air $(\gamma) = 1.4$

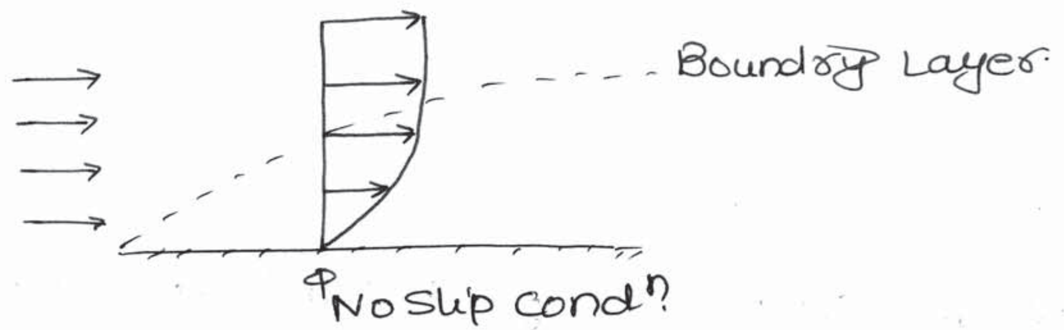
$$\boxed{K_{Adia} = \gamma\cdot P}$$

$$\boxed{\beta_{Adia} = \frac{1}{\gamma\cdot P}}$$

*Flow over a Flat plate

When a real fluid flows over a solid body, the fluid particle at the surface of the body flow with the same velocity as that of the surface to satisfy no slip condition. So the relative velocity of fluid particle at the surface of solid body is 0.

Away from the solid body, in the transverse direction the velocity of fluid particle increase gradually thus the velocity gradient exsit. in this region closed to boundary.



*Viscosity

The two adjacent layer of fluid resist the motion of each other. such a basic property of fluid is called viscosity.

Cause of viscosity

Liquids	Gases
Intermolecular force of attraction "Cohesive force"	Cohesion almost nil " Randomness of molecules.

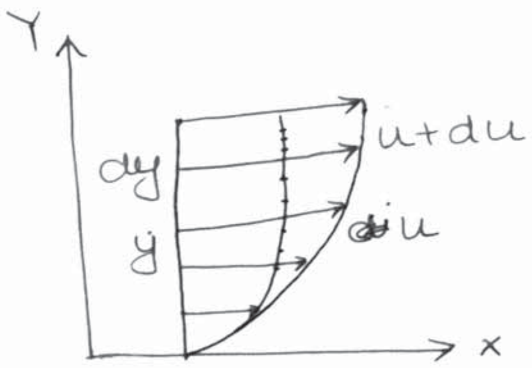
T = 20°C, P = 1 atm

$\mu_w = 0.001 \frac{N}{m^2} sec.$

$\mu_a = 0.000018 \frac{N}{m^2} \cdot S$

$\mu \rightarrow$ measurement of internal Resistance.

a) Angular Deformation

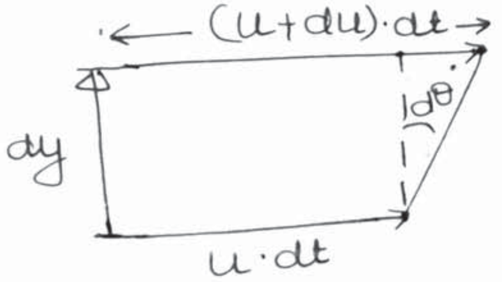


$$\tan\theta = \frac{du \cdot dt}{dy}$$

$d\theta$ = very-very small

$\tan\theta \approx d\theta$

In dt time interval



$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

↑ Rate of shear deformation

b) Newton's Law of viscosity

The viscous shear stress b/w the two adjacent layers at a distance y from the surface is

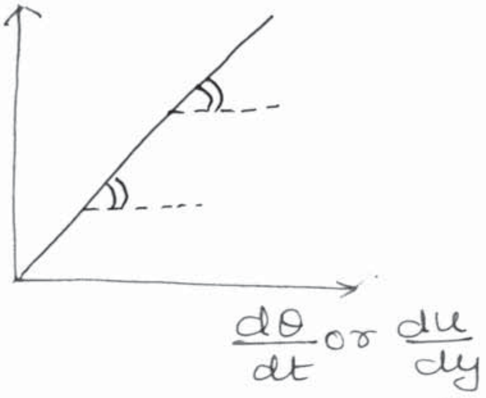
$$\boxed{\tau \propto \frac{d\theta}{dt}} \qquad \left[\frac{d\theta}{dt} = \frac{du}{dy}\right]$$

$$\tau \propto \frac{du}{dy}$$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

↓

dynamic viscosity
or
Coeff of viscosity.



Slope of curve = μ

Ex ⇒ Air, H_2O, Hg, petrol, kerosine etc.

* Fluids follow Newton's Law of viscosity is called Newtonian Fluid

c] Unit

Dynamic viscosity μ

$$\tau = \mu \frac{du}{dy}$$

$$\mu = \frac{\tau \times dy}{du}$$

In SI -

Unit of $\mu = \frac{N/m^2 \times m}{m/sec}$

$$\boxed{\mu = \frac{N}{m^2} \cdot sec}$$

$$\boxed{1 \frac{N}{m^2} \cdot sec = 1 \text{ pa} \cdot sec}$$

$$1 \frac{N}{m^2} \cdot sec = 1 \frac{kg}{m^2} \times \frac{m}{s^2}$$

$$\boxed{1 \frac{N}{m^2} \cdot sec = 1 \frac{kg}{m\text{-}sec}}$$

Dimension of $\mu = [ML^{-1}T^{-1}]$

IN C.G.S

$$\frac{gm}{cm\text{-}sec}$$

$$\boxed{1 \frac{gm}{cm\text{-}sec} = 1 \text{ poise}}$$

Relation

$$1 \text{ poise} = \frac{1 gm}{cm\text{-}sec} = \frac{10^{-3} kg}{10^{-2} m\text{-}sec}$$

$$\boxed{1 \text{ poise} = \frac{1}{10} \frac{kg}{m\text{-}sec}}$$

Kinematic viscosity

$$\boxed{\nu = \frac{\mu}{\rho}}$$

MKS

Unit of $\nu = \frac{\frac{kg}{m\text{-}s}}{kg/m^3}$

$$\boxed{\nu = m^2/sec}$$

C.G.S

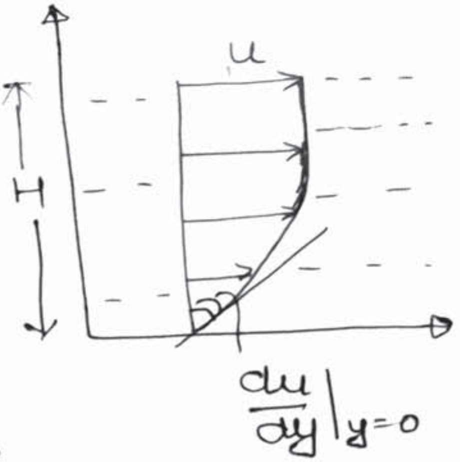
$$\boxed{1 \frac{cm^2}{sec} = 1 \text{ stoke}}$$

Relation

$$\boxed{1 \text{ stoke} = \frac{1}{10^4} m^2/sec}$$

* Lineaization of Newton Law of Viscosity

According to Newton's Law



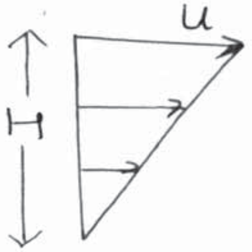
H = thickness of Flow

$$\tau = \mu \cdot \frac{du}{dy}$$

Shear Stress at wall (τ_0)

$$\tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0}$$ ← Velocity Profile

If H very-very small,
Then the velocity Profile can be treated as straight line



$$\tau_0 = \mu \frac{(U-0)}{H}$$

$$\tau_0 = \mu \cdot \frac{U}{H}$$

$$\left\{ \begin{aligned} \text{Drag Force } (F_D) &= \tau_0 \times A \\ &= \mu \frac{U}{H} \cdot A \end{aligned} \right\}$$

Concept →

i] Shear Stress acts on the upper plate in negative x-direction

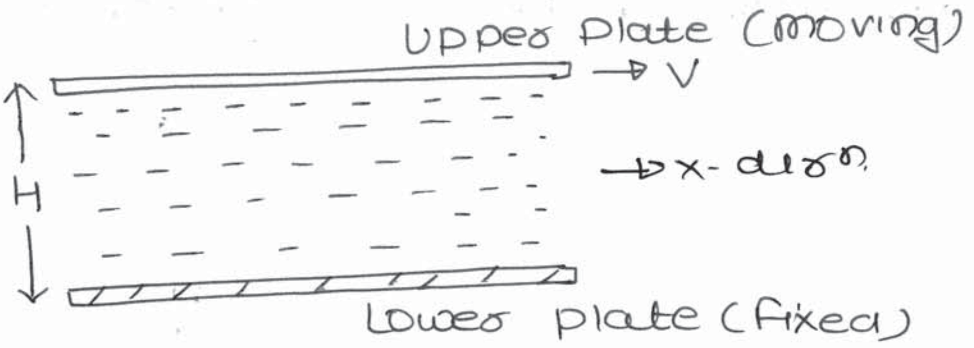
ii] Shear stress will act on the contacting layer with the ~~Lower~~ upper plate in positive dirn.

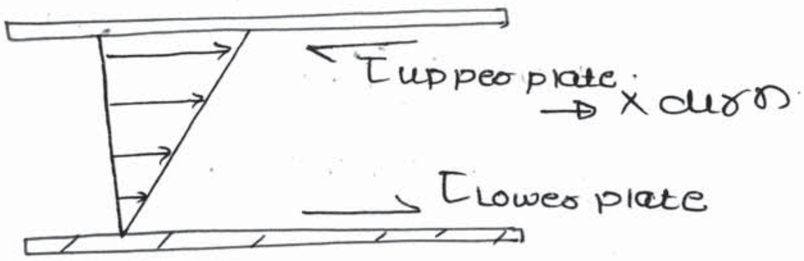
iii] Shear stress will act on the contacting Layer with the Lower plate in negative x-dirn.

iv] Shear stress will act on the Lower plate in positive x-dirn.

* *

H → very-very small



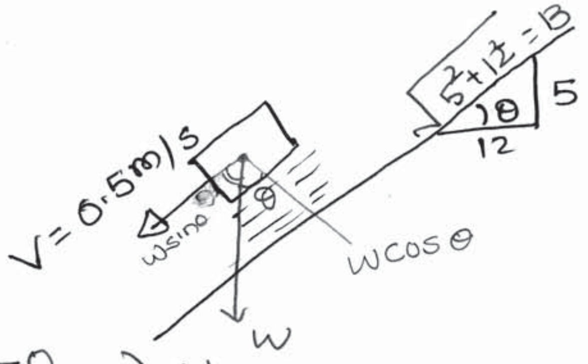


Q.18 W.B The viscosity of oil for the given fig.

h = 0.5 cm

A = 1 m²

W = 130 N



Soln

<u>NSL</u>

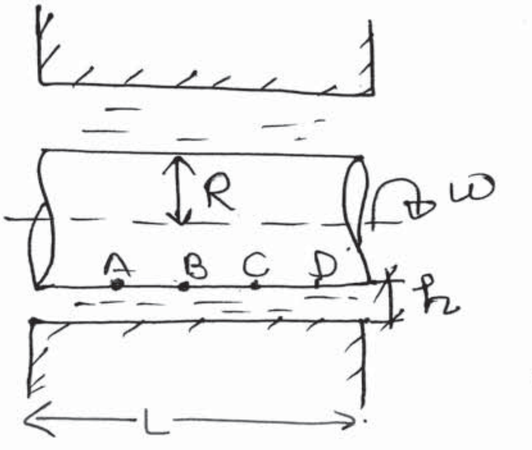
$$W\sin\theta = \tau \cdot A$$

$$130 \times \frac{5}{13} = \mu\left(\frac{0.5-0}{0.5\times10^{-2}}\right)\times 1$$

$$\boxed{\mu = 0.5\ \frac{N}{m^2}.\text{sec}}$$

Q.2 Determine power utilised in overcoming viscous Resistance

h = very-very small



$$\omega_A = \omega_B = \omega_C = \omega_D$$

$$V_A = V_B = V_C = V_D = \omega . R$$

$$\left\{\omega = \frac{2\pi N}{60}\right\}$$

$$P = T\cdot\omega$$

$$= F_D\cdot R\cdot\omega = \tau A$$

$$= \tau(2\pi RL)\cdot R\omega \Rightarrow \mu\frac{du}{dy}\times 2\pi R^2 L\omega$$

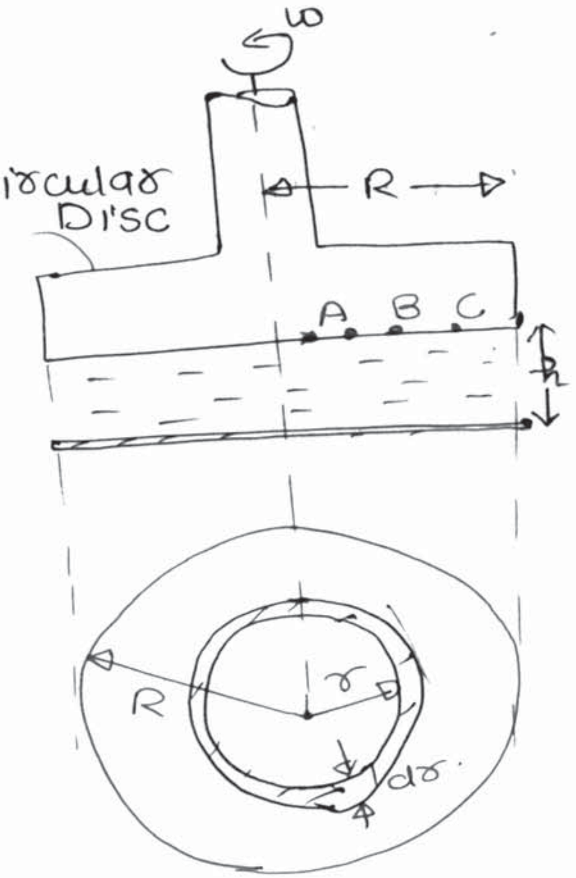
$$= \mu\left(\frac{\omega R-0}{h}\right)2\pi R^2\cdot L\cdot\omega$$

$$\boxed{P = \frac{2\pi\cdot\mu\omega^2 R^3\cdot L}{h}}$$

Que-3 | Determine.

A) Drag force on DISC

2) Torque required to maintain const. angular velocity ω



Sol

Sol^n a) Given

h = very-very small

$$\left\{\begin{matrix}\omega_A = \omega_B = \omega_C \\ V_A \neq V_B \neq V_C\end{matrix}\right\}$$

$$dF_D = \tau\cdot(2\pi r\cdot dr)$$

$$= \mu\left(\frac{\omega r-0}{h}\right)(2\pi r\,dr)$$

$$dF_D = \frac{2\pi\mu\omega r^2\cdot dr}{h}$$

Integrate $\quad F_D = \frac{2\pi\,\mu\cdot\omega}{h}\int_0^R r^2\cdot dr$

$$\boxed{F_D = \frac{2}{3}\frac{\pi\cdot\mu\omega R^3}{h}}$$

b) Differential Resistive force

$$dT = dF_D \cdot r$$

$$= \frac{2\pi \mu \omega r^2}{h} dr \cdot r$$

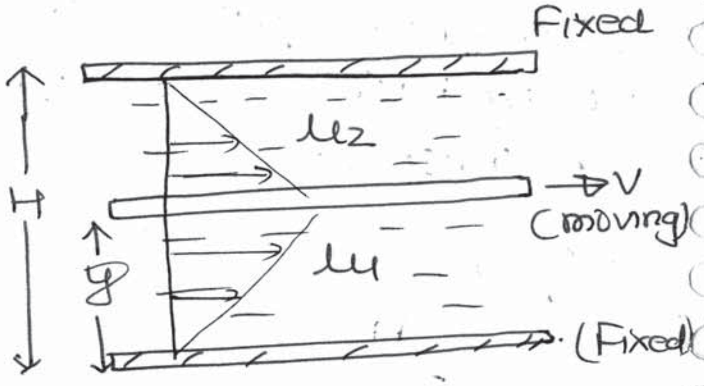
Integrate it $\quad T = \frac{2\pi \mu \omega}{h} \int_0^R r^3 dr.$

$$\boxed{T = \frac{\pi \mu \omega R^4}{2h}}$$

Q ⓐ Find y such that the Drag Force is same on both side of moving plate.

ⓑ Find y such that the total drag force is minm.

Sol^n H = very-very small



a) $F_{D_1} = F_{D_2}$

$\tau_1 A = \tau_2 \cdot A$

$$\mu_1\left(\frac{V-0}{y}\right) = \mu_2\left(\frac{V-0}{H-y}\right)$$

$$\frac{\mu_1 V}{y} = \frac{\mu_2 V}{H-y}$$

$$\mu_1 H = y(\mu_1+\mu_2)$$

$$\boxed{y = \frac{\mu_1 H}{\mu_1+\mu_2}}$$

b) $F_{DT} = F_{D_1} + F_{D_2}$

$= \tau_1 A + \tau_2 \cdot A$

$$= A\left\{\mu_1\left(\frac{V-0}{y}\right) + \mu_2\left(\frac{V-0}{H-y}\right)\right\}$$

$$= A\cdot V\left\{\frac{\mu_1}{y} + \frac{\mu_2}{H-y}\right\}$$

F_{DT} to be minm.

$$\frac{dF_{DT}}{dy} = 0$$

$$\frac{d}{dy} A\cdot V\left(\frac{\mu_1}{y} + \frac{\mu_2}{H-y}\right)$$

$$-\frac{\mu_1}{y^2} + \frac{\mu_2}{(H-y)^2} = 0$$

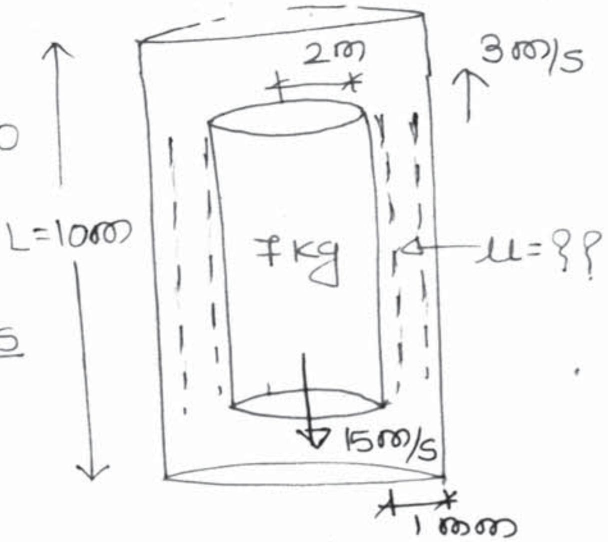
$$\frac{\sqrt{\mu_1}}{y} = \frac{\sqrt{\mu_2}}{H-y}$$

$$\boxed{\frac{H\cdot\sqrt{\mu_1}}{\sqrt{\mu_1}+\sqrt{\mu_2}} = y}$$

Q·05 $W = \tau \cdot A$

$$7 \times 9.81 = \mu \left[\frac{15-(-3)}{10^{-3}}\right] 2\pi(2) \times 10$$

$$\boxed{\mu = 3.035 \times 10^{-5} \frac{N}{m^2} \text{-Sec}}$$ Ans



Relative velocity $= 15-(-3)$

* Effect of Temperature on Viscosity

Liquid

Dynamic viscosity (μ_{Liq})

As T↑ ⇒ Intermolecular force of attraction decreases
= cohesive force decreases
$= \mu_{liq} \downarrow$

As T↑, μ_{Liq}↓

kinematic viscosity (ν_{liq})

As T↑, ρ_{liq}↓ (slightly)

$$\nu_{liq} = \frac{\mu_{liq} \downarrow (\text{Highly})}{\rho_{liq} \downarrow (\text{slightly})}$$

As T↑, ν_{liq}↓

Gases

Dynamic viscosity (μ_{gas})

As T↑ = Randomness of molcules increases
= Additional resistance in the path of molcules
$= \mu_{gas}$↑

As T↑, μ_{gas}↑

Kinematic viscosity (ν_{gas})

As T↑ $= \dfrac{\mu_{Gas}\uparrow}{\rho \downarrow}$ $P = \rho RT$

As T↑, ν_{gas}↑

Effect of Pressure on Viscosity

Liquid	Gases
Dynamic Viscosity (μ_{Liq})	Dynamic Viscosity (μ_{gas})
As P↑, μ_{liq} remain same	As P↑, μ_{gas} remain same
Kinematic Viscosity (ν_{Liq})	Kinematic viscosity (ν_{gas})
As P↑, ν_{Liq} remain same	As P↑, $\nu_{gas} = \frac{\mu_{gas}}{\rho\uparrow}$ As P↑, $\nu_{gas}\downarrow$

* Rheology

It is the branch of science which deals with the study of different types of fluid in flow

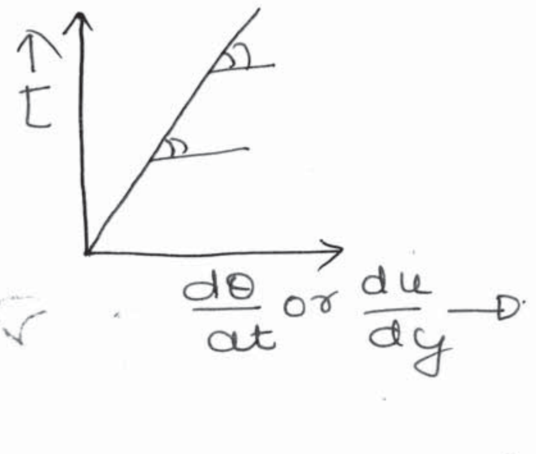
1] Newtonian Fluid

Obey's Newton's law of viscosity.

$$\tau \propto \frac{d\theta}{dt} \quad \left\{\frac{d\theta}{dt} = \frac{du}{dy}\right\}$$

$$\tau \propto \frac{du}{dy}$$

$$\boxed{\tau = \mu\frac{du}{dy}}$$ → slope of curve



Example – H_2O, Air, Hg, Petrol, Kerosene etc.

2| Non-Newtonian Fluids

Don't obey newton's Law of viscosity.

$$\boxed{\tau = A\left(\frac{du}{dy}\right)^n + B}$$

A → Consistency Index

n → Flow behaviour Index.

- Time independent Fluids.
- Time dependent Fluids.

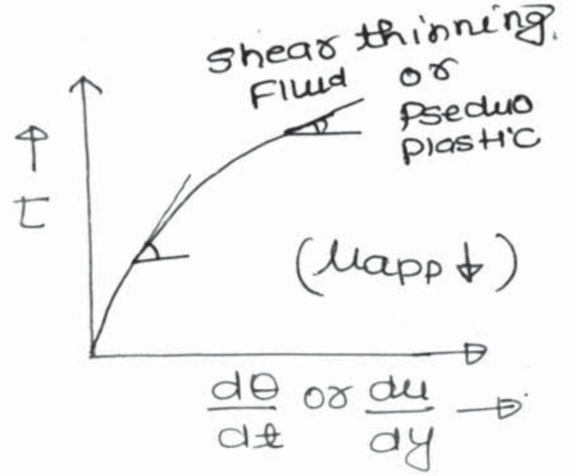
1 ⇒ Time Independent Fluids.

a) Pseudo Plastic Fluid

$n < 1$, $B = 0$

$$\tau = A\left(\frac{du}{dy}\right)^n$$

$$\tau = \underbrace{A\left(\frac{du}{dy}\right)^{n-1}}_{\mu_{apparent}} \times \frac{du}{dy}$$



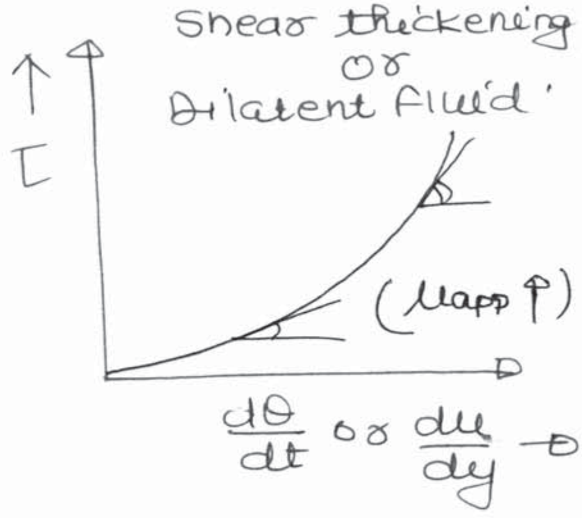
Ex ⇒ Blood, Mik etc.

b) Dilatent Fluid

$n > 1$, $B = 0$

$$\tau = A\left(\frac{du}{dy}\right)^n$$

$$\tau = \underbrace{A\left(\frac{du}{dy}\right)^{n-1}}_{\mu_{apparent}} \times \frac{du}{dy}$$



Example ⇒ concentrated solution of suger, Rice-Starch aqueous Suspension.

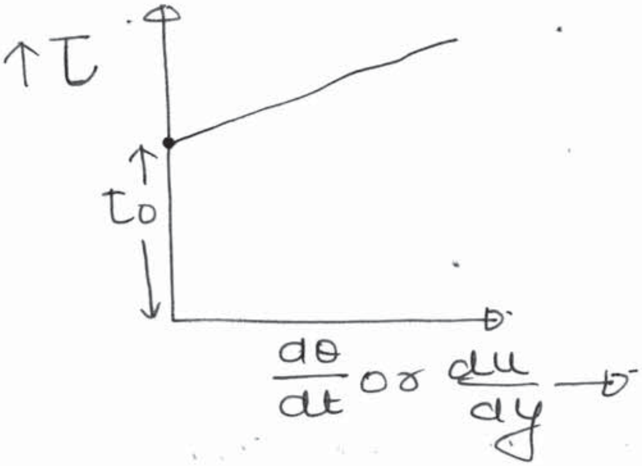
1) Bingham Plastic Fluid → [Ideal Plastic Fluid]

$$\boxed{\tau = A\left(\frac{du}{dy}\right)^{n} + B}$$

Here $n=1$

$B = \tau_0$ (Yield Stress)

Ex → Tooth paste, Drilling mud.



2] Time Dependent Fluid:

$$\boxed{\tau = A\left(\frac{du}{dy}\right)^{n} + B}$$

$B > 0$

i) Thixotropic fluid

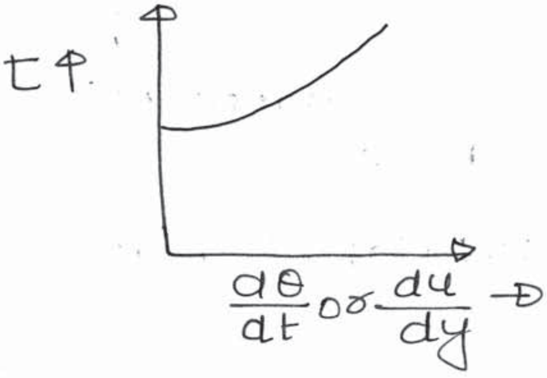
$n < 1$, $B > 0$

μ_{app} decreases w.r.t time under a given shear rate.

Ex → Some paints, printers ink etc.

ii) Rheopectic Fluid.

$n > 1$, $B > 0$



μ_{app} increases w.r.t time under a given shear rate

Ex → Gypsum suspension in H_2O.

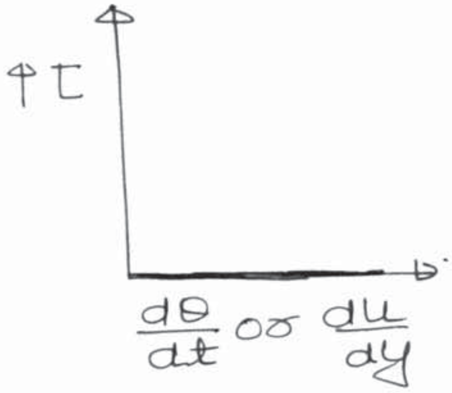
* Ideal Fluid

i) Frictionless

ii) Inviscid ($\mu = 0$)

iii) Incompressible

iv) No surface tension effect.



* Visco-Elastic Fluid

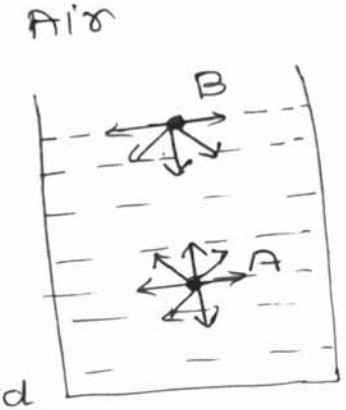
These Fluid having the property of elasticity up to certain limit.

Ex ⇒ Polymerized Fluid having draged reduction Features.

* Surface Tension (σ)

Molecule A (Inside the Liquid) $\vec{F}_A = 0$

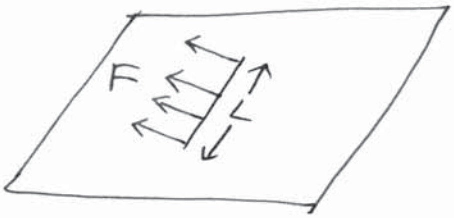
Molecule B (At the Surface) $\vec{F} \neq 0$ (Acts Inward Dirn)



Mathmatically

$$\boxed{\sigma = \frac{F}{L}}$$

F ⇒ effect of cohesive forces at the surface i.e Tensile Force.

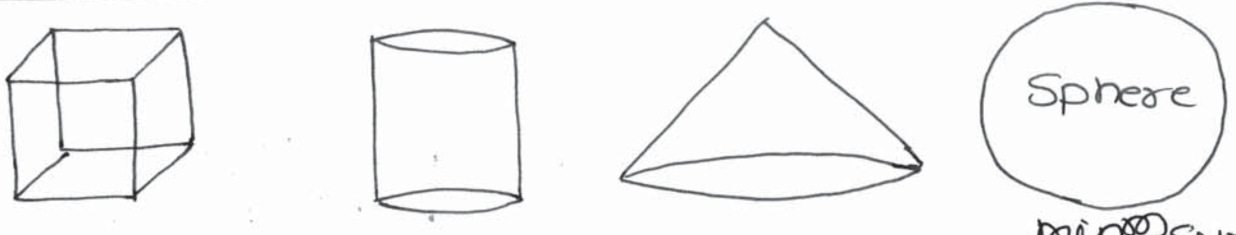


At 20°C

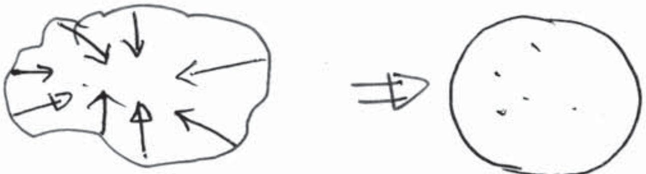
$$\boxed{\sigma_{w-a} = 0.073 \, N/m}$$

$$\boxed{T \uparrow = \text{Cohesive force} \downarrow = \sigma \downarrow}$$

In Genral take a Fix Volume.
Different Shape



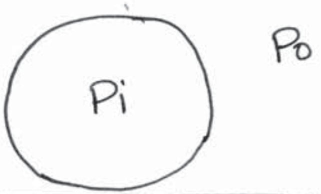
min^m surface area.



Raindrop

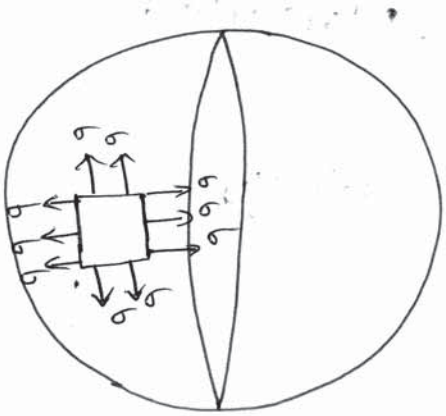
- The Liquid Drops are spherically shape.
- It is a property of Liquid by virtue which it try to minimize its surface area up to max^m extend.

In Genral

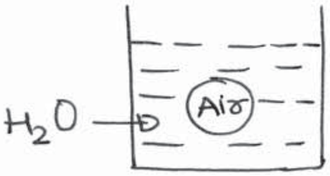


$$\boxed{\Delta P = P_i - P_o}$$

Pressure(P_i) > Press(P_o)

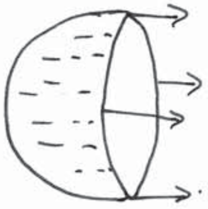


① Air bubble in H_2O



$$\boxed{\Delta P = \frac{2\sigma}{R}}$$

② Water Droplet



Radius = R.

Force Analysis

$$\Delta P(\pi R^2) = \sigma(2\pi R)$$

$$\boxed{\Delta P = \frac{2\sigma}{R}}.$$