ESE 2023

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Main Examination

Civil Engineering

Topicwise *Conventional* Solved Papers

Paper-II

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B. Singh (Ex. IES)

Director's Message

During the last few decades of engineering academics, India has witnessed geometric growth in engineering graduates. It is noticeable that the level of engineering knowledge has degraded gradually, while on the other hand competition has increased in each competitive examination including GATE and UPSC examinations. Under such scenario higher level efforts are required to take an edge over other competitors.

The objective of **MADE EASY books** is to introduce a simplified approach to the overall concepts of related stream in a single book with specific presentation. The topic-wise presentation will help the readers to study & practice the concepts and questions simultaneously.

The efforts have been made to provide close and illustrative solutions in lucid style to facilitate understanding and quick tricks are introduced to save time.

Following tips during the study may increase efficiency and may help in order to achieve success.

- Thorough coverage of syllabus of all subjects
- Adopting right source of knowledge, i.e. standard reading text materials
- Develop speed and accuracy in solving questions
- Balanced preparation of Paper-I and Paper-II subjects with focus on key subjects
- Practice online and offline modes of tests
- Appear on self assessment tests
- Good examination management
- Maintain self motivation
- Avoid jumbo and vague approach, which is time consuming in solving the questions
- Good planning and time management of daily routine
- Group study and discussions on a regular basis
- Extra emphasis on solving the questions
- Self introspection to find your weaknesses and strengths
- Analyze the exam pattern to understand the level of questions
- Apply shortcuts and learn standard results and formulae to save time

B. Singh (Ex. IES)
CMD, MADE EASY Group

ESE 2023 Main Examination

Civil Engineering
Conventional Solved Questions
Paper-II

Contents

1 Fluid Mechanics including Hydraulic Machines & OCF

Revised Syllabus of ESE : *Fluid Mechanics, Open Channel Flow, Pipe Flow: Fluid properties; Dimensional Analysis and Modeling; Fluid dynamics including flow kinematics and measurements; Flow net; Viscosity, Boundary layer and control, Drag, Lift, Principles in open channel flow, Flow controls. Hydraulic jump; Surges; Pipe networks.*

Hydraulic Machines and Hydro power: Various pumps, Air vessels, Hydraulic turbines – types, classifications & performance parameters; Power house – classification and layout, storage, pondage, control of supply.

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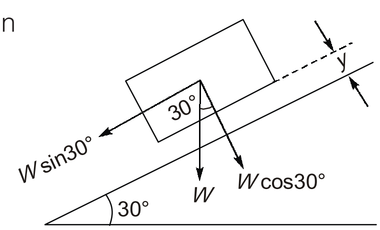
1. Fluid Properties

1.1 A plate with surface area of 0.4 m^2 and weight of 500 N slides down on an inclined plane at 30° to the horizontal at a constant speed of 4 m/s. If the inclined plane is lubricated with an oil of dynamic viscosity 2 poise, find the thickness of lubricant film.

[10 marks : 2006]

Solution:

Assuming linear relationship between shear stress developed in the lubricant and velocity gradient.



Let the thickness of the lubricating film be y

Surface Area of plate, $A = 0.4\ m^2$

Weight of plate, $W = 500$ N

Speed of sliding of plate, $V = 4$ m/s

Dynamic viscosity, $\mu = 2$ poise $= 0.2$ kg/m-s

The shear stress will be developed in the lubricant due to the component of the weight of the plate in the direction of motion. Let the component of weight in the direction of motion be F.

$\therefore \quad F = W\sin 30° = 500 \sin 30° = 250$ N

According to Newton's law of viscosity,

$$F = \frac{\mu AV}{y} \qquad \left[\because F = \tau A; \quad \tau = \mu \frac{du}{dy}\right]$$

$$\Rightarrow \quad 250 = \frac{0.2 \times 0.4 \times 4}{y}$$

$$\Rightarrow \quad y = 1.28 \times 10^{-3} \text{ m}$$

$$= 1.28 \text{ mm}$$

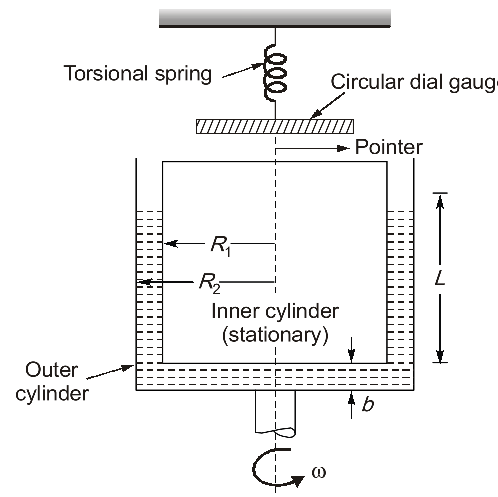
1.2 A rotating viscometer has two cylinders. The radius of inner fixed cylinder is R_1 and the radius of the outer rotating cylinder is R_2. This viscometer is used for the measurement of viscosity. Derive an expression for the viscosity in terms of the torque acting on the inner cylinder of height L, gap between the bottoms of the two cylinders b, and the angular speed ω (omega).

[9 marks : 2007]

Solution:

It consists of two co-axial cylinders, having radius R_1 and R_2 as shown in the figure. The very small space $(R_2 - R_1)$ is left in between the two. The space between them is filled with the liquid whose viscosity is to be determined.

The inner cylinder is suspended by a torsion wire on spring and it is held stationary. The outer cylinder is then rotated at a constant angular velocity. When the outer cylidner rotates, the torque generated by such rotation is transmitted by the thin liquid film to the inner stationary cylinder, which causes rotation of torsion wire. The rotation of wire can be measured by means of a circular dial attached to the wire and a fixed pointer.



From the previously obtained calibration curve between the torque and the rotation of torsion wire, the torque exerted on wire and hence on the inner cylinder, corresponding to the measured rotation of wire can be known.

Total torque, $T = T_1 + T_2$ ...(i)

T_1 = Torque due to side

T_2 = Torque due to bottom

Case-1:

Torque contributed from the sides, T_1

Circumferential velocity of the outer cylinder

$$V = \omega R_2$$

Clearance between the cylinders, $h = R_2 - R_1$

Assuming linear variation of velocity across the gap,

Velocity gradient $\dfrac{du}{dr} = \dfrac{V}{r} = \dfrac{\omega R_2}{R_2 - R_1}$

Shear stress, $\tau = \mu \dfrac{du}{dr} = \dfrac{\mu\omega R_2}{R_2 - R_1}$

Shear force, $F_s = \tau \times 2\pi R_1 \times L$

$\therefore$ $T_1 = F_s \times R_1$

$\Rightarrow$ $T_1 = \tau \times 2\pi R_1 \times L \times R_1$

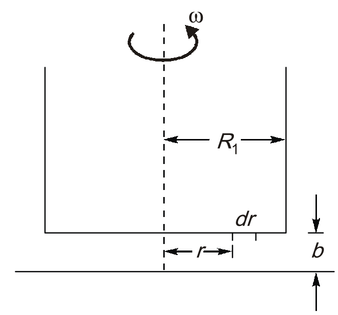
$\Rightarrow$ $T_1 = \dfrac{\mu\omega R_2}{(R_2 - R_1)} \times 2\pi R_1^2 L$

$$= \frac{2\pi\mu\omega R_1^2 R_2 L}{R_2 - R_1} \quad \text{...(ii)}$$

Case-2:

Torque contributed from the bottom (T_2)

Consider an element of inner cylinder of width 'dr' at a radial distance r.



Velocity at this radius, $v = r\omega$

Assuming linear variation of velocity with depth in the gap 'b'

Shear stress, $$\tau = \frac{\mu v}{b} = \frac{\mu r\omega}{b}$$

Torque of the element, $$dT_2 = \frac{\mu r\omega}{b}(2\pi r\, dr)r = \frac{\mu\omega}{b}2\pi r^3 dr$$

Total torque on the cylinder, $$T_2 = \int_0^{R_1}\frac{\mu\omega}{b}2\pi r^3 dr$$

$$\Rightarrow \quad T_2 = \frac{\mu\omega}{b}2\pi\left[\frac{r^4}{4}\right]_0^{R_1} = \frac{\mu\omega}{b}\frac{2\pi R_1^4}{4} = \frac{\pi\mu\omega R_1^4}{2b} \quad \text{... (iii)}$$

Total torque, $$T = T_1 + T_2$$

$$T = \frac{2\pi\mu\omega R_1^2 R_2 L}{R_2 - R_1} + \frac{\pi\mu\omega R_1^4}{2b}$$

$$T = \left(\frac{2\pi R_1^2 . R_2 L}{R_2 - R_1} + \frac{\pi R_1^4}{2b}\right)\omega\mu$$

$$\mu = \frac{T}{\omega\left(\frac{2\pi R_1^2 R_2 L}{R_2 - R_1} + \frac{\pi R_1^4}{2b}\right)}$$

1.3 Through a very narrow gap of height 'h' a thin plate of large extent is pulled at a velocity V. On one side of the plate oil of viscosity μ_1 and on other side oil of viscosity μ_2. Calculate the position of plate so that:

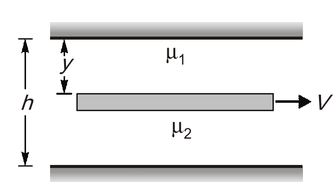
(i) The shear force on two sides of plate is equal

(ii) The pull required to drag the plate is minimum

[10 marks: 2008]

Solution:

Let y be the distance of the thin plate from the top surface. Assuming linear relationship between shear stress developed and the velocity gradient.



(i) **Shear stress developed on the top portion** is given by,

$$\tau_1 = \mu_1\frac{du}{dy}$$

$$\Rightarrow \quad \tau_1 = \mu_1 \times \frac{V}{y}$$

Shear stress developed on the bottom portion is given by

$$\tau_2 = \mu_2 \times \frac{V}{h-y}$$

If A is the area of thin plate, then shear force on the top and bottom portion,

$$F_1 = \tau_1 \times A \text{ and } F_2 = \tau_2 \times A$$

But $$F_1 = F_2$$
$$\Rightarrow \quad \tau_1 \times A = \tau_2 \times A$$
$$\Rightarrow \quad \mu_1 \times \frac{V}{y} = \mu_2 \times \frac{V}{h-y}$$
$$\Rightarrow \quad \mu_1(h-y) = \mu_2 y$$
$$\Rightarrow \quad y(\mu_1 + \mu_2) = \mu_1 h$$
$$\Rightarrow \quad y = \frac{\mu_1 h}{\mu_1 + \mu_2} \text{ (Ans.)}$$

(ii) The pull required to drag the plate = Total shear force

$$\Rightarrow \quad F = F_1 + F_2$$
$$\Rightarrow \quad F = \tau_1 A + \tau_2 A$$
$$\Rightarrow \quad F = \mu_1 \times \frac{V}{y} \times A + \mu_2 \times \frac{V}{h-y} \times A = \left[\frac{\mu_1}{y} + \frac{\mu_2}{h-y}\right] VA$$

For F to be minimum, $$\frac{dF}{dy} = 0$$
$$\Rightarrow \quad -\frac{\mu_1}{y^2} + \frac{\mu_2}{(h-y)^2} = 0$$
$$\Rightarrow \quad \frac{y^2}{(h-y)^2} = \frac{\mu_1}{\mu_2}$$
$$\Rightarrow \quad \frac{y}{h-y} = \frac{\sqrt{\mu_1}}{\sqrt{\mu_2}}$$
$$\Rightarrow \quad \left(\sqrt{\mu_1} + \sqrt{\mu_2}\right) y = \sqrt{\mu_1} h$$
$$\Rightarrow \quad y = \frac{\sqrt{\mu_1} h}{\sqrt{\mu_1} + \sqrt{\mu_2}} \text{ (Ans.)}$$

1.4 The velocity distribution for flow over a plate is given by

$$u = 2y - y^2$$

in which u is the velocity in ms^{-1} at a distance y metres from the plate. Determine the shear stress in Nm^{-2} at the boundary and at 0.2 m from it. Dynamic viscosity of fluid is 0.9 Ns/m^2.

[4 marks : 2013]

Solution:

Given $$u = 2y - y^2$$
and $$\mu = \text{Dynamic viscosity of fluid} = 0.9 \text{ Ns/m}^2$$

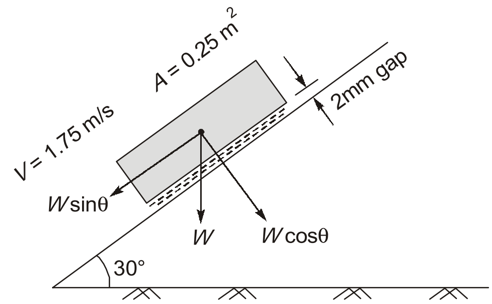
$$\text{Shear stress } (\tau) = \mu \frac{\partial u}{\partial y} = \mu(2 - 2y)$$

$$\therefore \quad \tau|_{y = 0.2\text{ m}} = 0.9(2 - 2 \times 0.2) = 1.44 \text{ N/m}^2$$
and $$\tau_{1_{y=0}} = 0.9 \times 2 = 1.8 \text{ N/mm}^2$$

1.5 A rectangular plate of 0.50 m × 0.50 m dimensions weighing 500 N slides down an inclined plane making 30° angle with the horizontal, at a velocity of 1.75 m/s. If the 2 mm gap between the plate and the inclined surface is filled with a lubricating oil, find its viscosity and express it in poise as well as in Ns/m^2.

[4 marks : 2014]

Solution:



Area of plate, $A = 0.50 \times 0.50 = 0.25\,m^2$

Weight of plate, $W = 500\,N$

$$W \sin\theta = F_{drag}$$

$$500 \sin 30° = \tau \cdot A$$

$$\Rightarrow \quad \mu \frac{du}{dy} A = 500 \sin 30°$$

$$\Rightarrow \quad \mu \frac{(V-0)}{2 \times 10^{-3}} \times 0.25 = 500 \sin 30°$$

$$\mu = \frac{500 \sin 30° \times 2 \times 10^{-3}}{1.75 \times 0.25} = 1.143\ \text{N-s/m}^2$$

Since; $1\ \text{Poise} = 10^{-1}\ \text{N-s/m}^2$

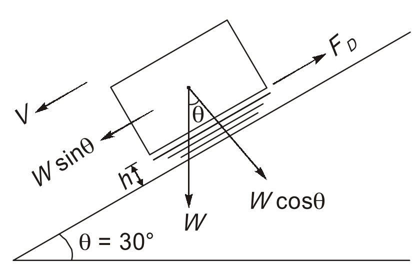
$$\Rightarrow \quad 1 \frac{\text{N-s}}{\text{m}^2} = 10\ \text{Poise}$$

$$\therefore \quad \mu = 11.43\ \text{poise or } 1.143\ \text{N-s/m}^2$$

1.6 A rectangular plate of 0.5 m × 0.5 m dimensions, weighing 500 N slides down an inclined plane making 30° angle with the horizontal at a velocity of 1.75 m/s. If the 2 mm gap between the plate and inclined surface is filled with a lubricating oil, find its viscosity in poise.

[6 marks : 2020]

Solution:



Force analysis in direction of motion

$$F_D = W \sin\theta$$

$$\tau A = 500 \sin 30° \quad \text{...(i)}$$

$\because$ Shear stress, $\tau = \mu \dfrac{du}{dy}$

{Since the gap is very-very small so velocity variation is considered as linear.}

$$\tau = \mu \frac{V-0}{h}$$

$$\tau = \mu \frac{V}{h}$$

By eq. (i) $\mu \frac{V}{h} A = 500 \sin 30°$

$$\mu \frac{(1.75)}{0.002} \times 0.5 \times 0.5 = 500 \sin 30°$$

$$\mu = 1.143 \text{ Ns/m}^2$$

$$\mu = 11.43 \text{ Poise}$$

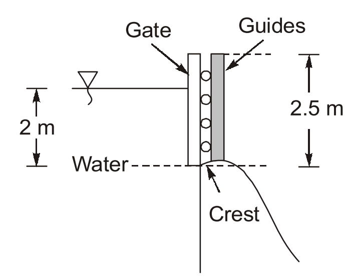
2. Manometry and Hydrostatic Forces

2.1 A vertical lift gate 5 m × 2.5 m size weighing 0.5 tonnes slides along guides (coefficient of friction is 0.25) fitted on the side walls of an over flow spillway and its crest. What force will have to be exerted at the hoisting mechanism to lift the gate when the head of water over the crest is 2 m.

[10 marks : 1998]

Solution:

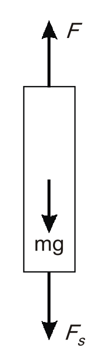
Let the normal reaction at the guides be R. The normal reaction will be equal to the total hydrostatic pressure acting on the gate i.e.



$$\therefore \quad R = wA\bar{x}$$

where w is specific weight of the liquid, $\bar{x}$ is the depth of centroid of plate below the free surface of the liquid and A is the cross-sectional area of plate.

$$\Rightarrow \quad R = 1000 \times 9.81 \times (5 \times 2) \times \left(\frac{2}{2}\right) = 98100 \text{ N}$$



Now, when the gate is lifted upwards, the frictional force F_s will be developed which acts in vertically downward direction.

$$\therefore \quad F_s = \mu R = 0.25 \times 98100 = 24525 \text{ N}$$

If the force required to lift the gate is F, then it is opposed by the self weight of gate and frictional force F_s

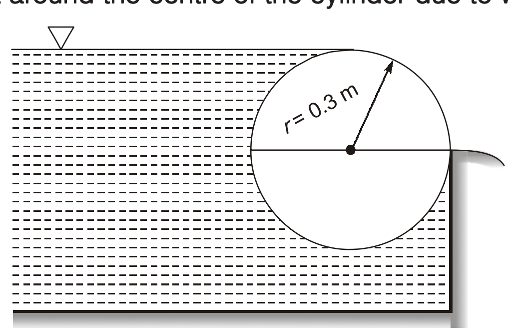
$$\therefore \quad F = F_s + mg = 24525 + (0.5 \times 1000 \times 9.81)$$
$$= 29430 \text{ N} = 29.43 \text{ kN}$$

2.2 A cylinder of radius 0.3 m is located in water as shown. The cylinder and the wall are smooth. For a 1.5 m length of cylinder, find

(i) its weight,

(ii) the resultant force exerted by the wall on the cylinder,

(iii) the resultant moment around the centre of the cylinder due to water forces on the cylinder.



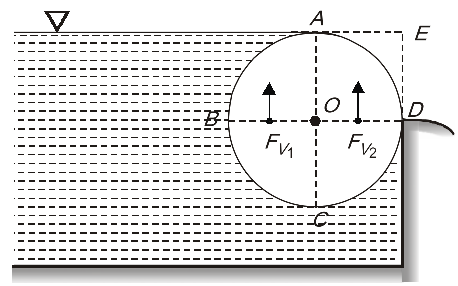
[15 marks : 1998]

Solution:

(i) **Since the cylinder and wall are smooth, the weight of the cylinder is equal to the vertical component of the hydrostatic pressure acting on the cylinder.**

But we know that vertical component of hydrostatic force is equal to the weight of the fluid contained in the curved surface upto free surface.

$\therefore$ F_{V_1} = weight of water in portion ABC acting vertically upwards through the C.G of area *ABC*



$\Rightarrow$ F_{V_1} = 9.81 × volume of semicircle ABC

$$= 9.81 \times \frac{\pi \times (0.3)^2}{2} \times 1.5$$

$$= 2.08 \text{ kN}$$

Centre of gravity of semicircle *ABC*

$$= \frac{4r}{3\pi} = \frac{4 \times 0.3}{3\pi} = 0.1273 \text{ m}$$

F_{V_2} = weight of water in the portion *ACDE*

acting vertically upwards at the centroid of the area *ACDE*

$\Rightarrow$ F_{V_2} = 9.81 × volume of quarter circle *COD* + 9.81 × volume of square *AODE*

$$= 9.81 \times \frac{1}{4} \times \pi \times (0.3)^2 \times 1.5 + 9.81 \times 0.3 \times 0.3 \times 1.5$$

$$= 1.0401 + 1.3244 = 2.3645 \text{ kN}$$

If x is the distance of centre of gravity of *ACDE* from *AC*, then

$$2.3645x = 1.0401 \times \frac{4 \times 0.3}{3\pi} + 1.3244 \times \frac{0.3}{2}$$

$\Rightarrow$ x = 0.14 m

$\therefore$ Weight of cylinder = $F_{V_1} + F_{V_2}$ = 2.08 + 2.3645 = 4.4445 kN

(ii) **The horizontal component of the hydrostatic force exerted on the cylinder will consist of:**

(a) F_{H_1} acting horizontally from left to right on the vertical projection of the curved surface *AB*

(b) F_{H_2} acting horizontally from left to right on the vertical projection of the curved surface *BC*

(c) F_{H_3} acting horizontally from right to left on the vertical projection of the curved surface *CD*

Now F_{H_2} and F_{H_3} are equal and opposite. Thus the total horizontal force will be F_{H_1} acting at the centre of pressure of the vertical projection of curved surface *AB*.

$$F_{H_1} = 9.81 \times 0.3 \times 1.5 \times \frac{0.3}{2} = 0.6622 \text{ kN} \qquad [\text{using } F_H = wA\bar{x}]$$

$$\text{Centre of pressure} = \bar{x} + \frac{I_G}{A\bar{x}} = \frac{0.30}{2} + \frac{1.5 \times (0.3)^3 \times 2}{12 \times 1.5 \times 0.3 \times 0.3} = 0.15 + 0.05 = 0.20 \text{ m}$$

Thus F_{H_1} acts at a depth of 0.20 m from free surface or at a height of 0.1 m above the centre of cylinder (*O*).

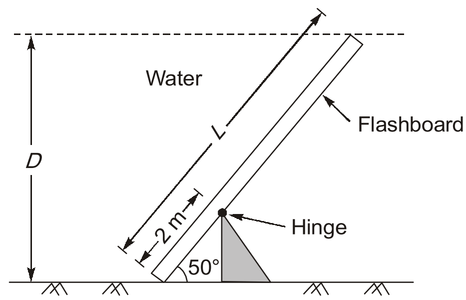
The resultant force exerted by the wall on the cylinder,

$$R = \sqrt{(F_{H_1})^2 + (F_{V_1} + F_{V_2})^2} = \sqrt{(0.6622)^2 + (4.4445)^2} = 4.4935 \text{ kN}$$

(iii) The resultant moment about the centre of cylinder due to hydrostatic forces will be given by

$$M = F_{H_1} \times 0.1 + F_{V_1} \times 0.1273 - F_{V_2} \times 0.14$$
$$= (0.6622 \times 0.1) + (2.08 \times 0.1273) - (2.3645 \times 0.14) = -0.000026 \text{ kN-m}$$
$$\approx 0$$

2.3 Find the depth of water required to topple the rectangular flashboard and reaction at the hinge of the flashboard shown in figure.



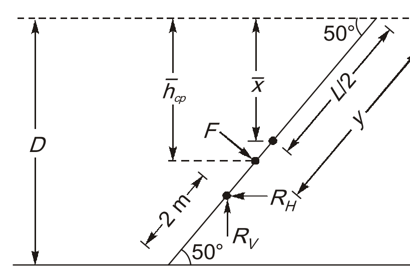
[10 marks : 2006]

Solution:

Let the centre of gravity of the flashboard be at a distance $\bar{x}$ from the free surface.

Assuming unit width of the flashboard perpendicular to the plane of paper

Hydrostatic force on the flashboard is given by



$$F = wA\bar{x} = w(L \times 1) \times \bar{x} = wL\bar{x}$$

$$\sin 50° = \frac{D}{L} = \frac{\bar{x}}{L/2}$$

$$\therefore \quad \bar{x} = \frac{L}{2}\sin 50° \text{ and } D = L\sin 50°$$

$$\therefore \quad F = wL\bar{x} \quad [\because \text{Area} = L \times 1, \text{ for unit width}]$$

$$\Rightarrow \quad F = wL \times \frac{L}{2}\sin 50° = \frac{wL^2}{2}\sin 50°$$

The hydrostatic force F will act at the centre of pressure $(\bar{h})$.

$$\therefore \quad \bar{h}_{cp} = \bar{x} + \frac{I_G}{A\bar{x}}\sin^2\theta$$

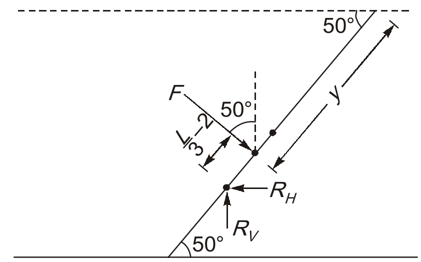
$$\bar{h}_{cp} = \frac{L}{2}\sin 50° + \frac{1 \times L^3 \times 2}{12 \times (L \times 1) \times L\sin 50°}\sin^2 50° \quad \left[\because I_G = \frac{bd^3}{12} = \frac{1 \times L^3}{12}\right]$$

$$= \frac{L}{2}\sin 50° + \frac{L}{6}\sin 50° = \frac{2}{3}L\sin 50°$$

Now, we have

$$\sin 50° = \frac{\bar{h}_{cp}}{y}$$

$$\Rightarrow \quad y = \frac{\frac{2}{3}L\sin 50°}{\sin 50°} = \frac{2}{3}L$$



Thus the perpendicular distance of the line of action of the hydrostatic force F from the hinge is given by

$$\text{Lever Arm} = L - \frac{2}{3}L - 2 = \frac{L}{3} - 2$$

Taking the moment of all the forces about the hinge, we get

$$F\left(\frac{L}{3} - 2\right) = 0$$

$$\Rightarrow \quad \frac{wL^2}{2}\sin 50^\circ\left(\frac{L}{3} - 2\right) = 0$$

$$\Rightarrow \quad L = 6 \text{ m}$$

$$\therefore \quad D = L \sin 50^\circ = 6 \sin 50^\circ = 4.6 \text{ m}$$

Now for equilibrium,

$$R_H = F\sin 50^\circ = \frac{wL^2}{2}\sin 50^\circ \times \sin 50^\circ = \frac{9810 \times (6)^2}{2} \times \sin^2 50^\circ = 103.62 \text{ kN}$$

$$R_V = F\cos 50^\circ = \frac{wL^2}{2}\sin 50^\circ \times \cos 50^\circ = \frac{9810 \times (6)^2}{2} \times \sin 50^\circ \times \cos 50^\circ = 86.95 \text{ kN}$$

Resultant Reaction, $R = \sqrt{R_H^2 + R_V^2} = \sqrt{(103.62)^2 + (86.95)^2} = 135.27 \text{ kN}$

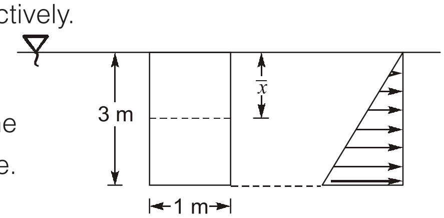
2.4 **Determine the total pressure on a plane rectangular plate 1 m wide and 3 m deep when its upper edge is horizontal and coincides with water surface and plate is held perpendicular to water surface.**

[2 marks : 2010]

Solution:

Let the width and depth of the rectangular plate be b and d respectively.

Total pressure on the rectangular plate will be given as



$$P = \gamma A \bar{x}$$

where γ is the unit weight of water, A is the area of rectangular plane surface and $\bar{x}$ is the distance of centre of gravity from water surface.

$$\therefore \quad P = \gamma \times b \times d \times \bar{x}$$

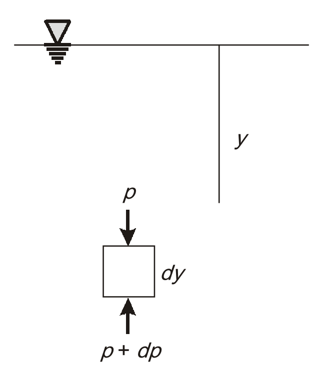
$$= 9810 \times 1 \times 3 \times \frac{3}{2} \qquad \left[\because \bar{x} = \frac{d}{2}\right]$$

$$= 44145 \text{ N} = 44.145 \text{ kN}$$

2.5 **Show that the hydraulic pressure remains invariant in a horizontal plane parallel to free surface.**

[4 marks : 2010]

Solution:



Consider an element of area dA, is y height below the free surface level, in a fluid of density ρ, hence for equilibrium

$$p\,dA + \text{Weight of liquid in a volume of } dA \cdot dy = (p + dp)dA$$

$$p\,dA + \rho g\,(dA\,dy) = (p + dp)dA$$

$$\rho g \cdot dA\,dy = dp \cdot dA$$

$$\frac{dp}{dy} = \rho g$$

$$p = \rho g y + \text{constant}$$

$$p \propto y$$

Hydrostatic pressure $\propto$ Depth

Hence, hydrostatic pressure varies only in vertical direction. Hence at a particular depth below the free surface hydrostatic pressure will remain same in a horizontal plane.

2.6 A 45° sector gate is located on the crest of spillway. The water is upto the mid-point of the gate when closed. The width of the gate is 10 m. The radius of the sector gate is 2 m. Determine the hydrostatic force on the gate. Mass density 1000 kg/m³, g = 9.79 ms⁻².

[10 marks : 2011]

Solution:

Given: Radius of the sector gate $= R = 2$ m

Width of the gate $= L = 10$ m

Height of water, h above the bottom tip of the gate

$$\Rightarrow \quad \sin\theta = \frac{h}{R}$$

$$\Rightarrow \quad h = R\sin\theta = 2\sin 22.5 = 0.765 \text{ m}$$

Hydrostatic force, $P = \sqrt{P_H^2 + P_V^2}$ ...(i)

$$P_H = \gamma_w A\bar{x}$$

Here A is area of vertical projection of the gate and $\bar{x}$ is the C.G. of the vertical projection from top.

$$P_H = \gamma_w \cdot (h \times L) \times \frac{h}{2} = (9.79 \times 1000)(0.765 \times 10) \times \frac{0.765}{2}$$

$$= 28646.76 \text{ N} = 28.65 \text{ kN}$$

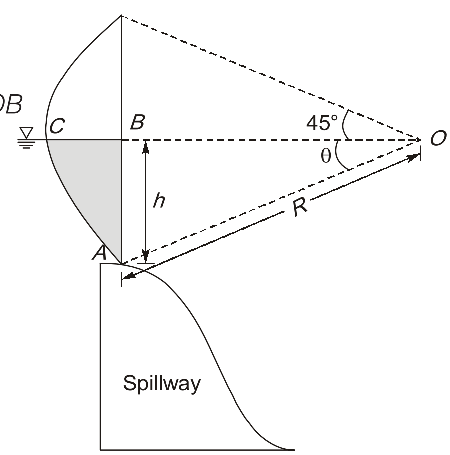
P_v = Vertical component of the water pressure

= Weight of imaginary volume of water *ABC*.

Area *ABC* = Area of sector *AOC* – Area of triangle *AOB*

$$= \frac{\pi \times 2^2}{360} \times 22.5 - \frac{1}{2} \times 0.765 \times \frac{0.765}{\tan 22.5}$$

$$= 0.0794 \text{ m}^2$$



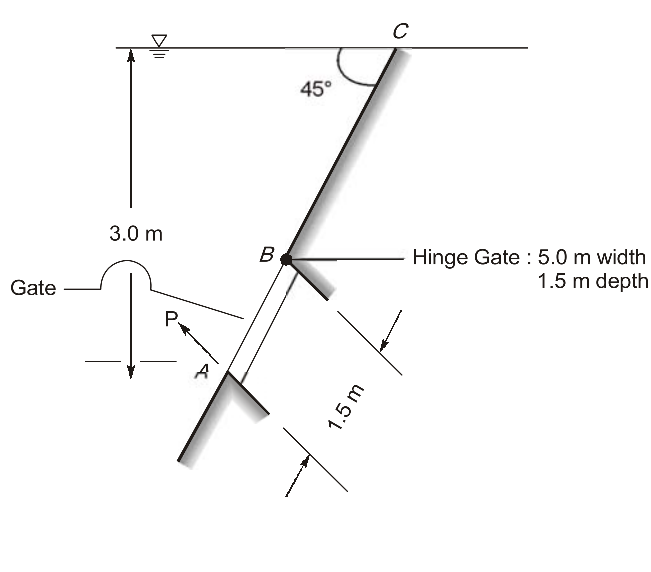
$$\therefore \quad P_v = (0.0794 \times 10 \times 9.79 \times 1000)$$

$$= 7773.26 \text{ N} = 7.773 \text{ kN}$$

From (i) Hydrostatic Force,

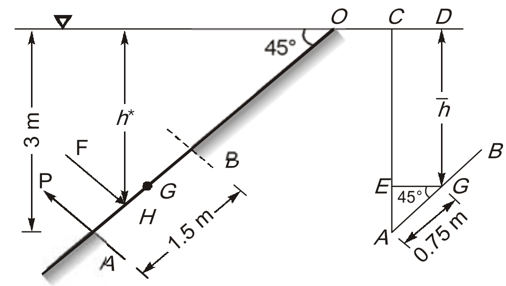
$$P = \sqrt{28.65^2 + 7.773^2} = 29.69 \text{ kN}$$

2.7 A rectangular gate of width 5 m and depth 1.5 m is installed to control the discharge as shown in figure below. The end '*B*' is hinged. Determine the force normal to the gate applied at '*A*' to open it.



[6 marks : 2012]

Solution:



Given: A = Area of gate = $1.5 \times 5 = 7.5$ m^2

Depth of C.G. of gate from free surface of water = $\bar{h}$

$$= DG = AC - AE$$

$$= 3 - AG \sin 45°$$

$$= 3 - 0.75 \times \frac{1}{\sqrt{2}}$$

$$= 2.4697 \text{ m}$$

The total pressure force (F) acting on the gate,

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 7.5 \times 2.4697 = 181708.18 \text{ N} = 181.71 \text{ kN}$$

This force is acting at point H where depth of H from free surface is given by

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where, $$I_G = \text{M.O.I of gate} = \frac{bd^3}{12} = \frac{5 \times 1.5^3}{12} = 1.40625 \text{m}^4$$

∴ Depth of centre of pressure $$h^* = \frac{1.40625 \times \sin^2 45°}{7.5 \times 2.4697} + 2.4697$$

$$= 0.03796 + 2.4697 = 2.508 \text{ m}$$

As from figure $$\sin 45° = \frac{h^*}{OH}$$

∴ Distance, $$OH = \frac{h^*}{\sin 45°} = \frac{2.508}{\frac{1}{\sqrt{2}}} = 3.547 \text{m}$$

Distance, $$AO = \frac{3}{\sin 45°} = 4.243 \text{m}$$

∴ Distance, $AH = AO - OH = 4.243 - 3.547 = 0.696$ m

∴ Distance $BH = AB - AH = 1.5 - 0.696 = 0.804$ m

Taking the moments about the hinge B

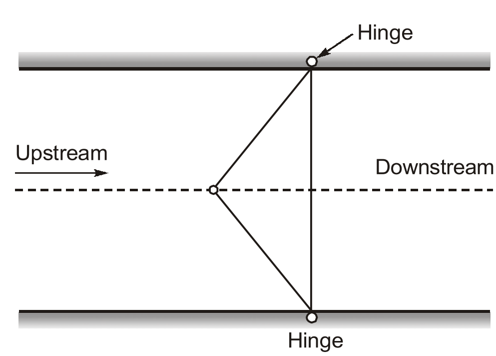
$$P \times AB = F \times (BH)$$

∴ $$P \times 1.5 = 181708.18 \times 0.804$$

⇒ $$P = \frac{181708.18 \times 0.804}{1.5} = 97395.585 \text{ N} = 97.396 \text{ kN}$$

2.8 The gates of lock are 5 cm wide by 6 m and when closed, at an angle of 120°. Each gate is held on by two hinges placed at the top and bottom of the gate. If the water levels are 6 m and 4.5 m on the upstream and downstream sides respectively, determine the magnitude of the forces on the hinges due to the water pressure.

[20 marks : 2013]



Lock gate in plan

Solution:

Width of the gates of lock = 5 cm

Although 5 cm width is practically not possible, it may be due to printing error in exam. Assuming data given is correct.

$\therefore$ Width, $W = 5$ cm

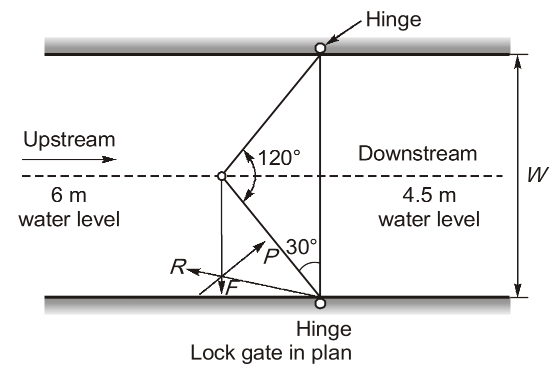


Fig. (i)

$\therefore$ Width of each gate $= \dfrac{5 \times 10^{-2}}{2\cos 30°} = 0.02887$ m

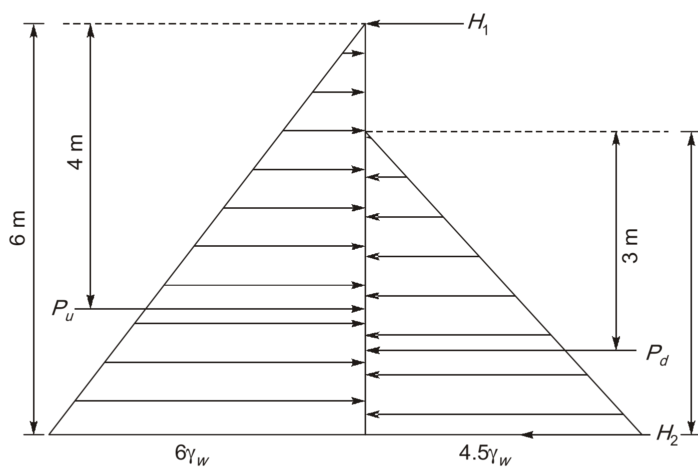


Fig. (ii)

Total pressure on the upstream face of the gate is

$$P_u = \rho g\bar{h}A = 1000 \times 9.81 \times \frac{6}{2} \times [6 \times 0.02887]$$

$$= 5097.8646 \text{ N} = 5.0978 \text{ kN}$$

The depth of the centre of pressure on the upstream face is given by

$$\bar{h}_u = \bar{h} + \frac{I_{CG}}{A\bar{h}} = 3 + \frac{\frac{1}{12} \times 0.02887 \times 6^3}{(0.02887 \times 6) \times 3} = 4 \text{ m}$$

Total pressure on the downstream face of the gate is

$$P_d = \rho g\bar{h}A = 1000 \times 9.81 \times \frac{4.5}{2} \times (0.02887 \times 4.5)$$

$$= 2867.5488 \text{ N} = 2.86755 \text{ kN}$$

Depth of centre of pressure on downstream face is

$$\bar{h}_d = \bar{h} + \frac{I_{CG}}{A\bar{h}} = 2.25 + \frac{\frac{1}{12} \times 0.02887 \times 4.5^3}{(0.02887 \times 4.5) \times 2.25} = 3 \text{ m}$$

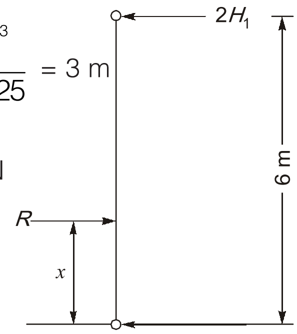


Fig. (iii)

Resultant water pressure on each gate

$$P = P_u - P_d = 5.0978 - 2.8675 = 2.2303 \text{ kN}$$

If x is the height of the point of application ofthe resultant water pressure on the gate, then

$$P \times x = P_u \times (6 - \bar{h}_u) - P_d \times (4.5 - \bar{h}_d)$$

$$\therefore \quad 2.2303\, x = 5.0978 \times (6 - 4) - 2.8675\, (4.5 - 3)$$

$$\therefore \quad x = 2.643 \text{ m}$$

Consider Free Body Diagram from fig. (i), we get

$$F = R$$

Also, $$F \sin 30 + R \sin 30 = P$$

$$\therefore \quad R = \frac{P}{2 \sin 30} = P = 2.2303 \text{ kN}$$

Now, from fig. (iii)

$\Sigma F_x = 0;$ $$R = 2H_1 + 2H_2$$

$\Sigma M_{H_2} = 0;$ $$R \times x = 2H_1 \times 6$$

$\therefore$ Force on one top hinge, $$H_1 = \frac{2.2303 \times 2.643}{6} = 0.4912 \text{ kN}$$

$$\therefore \quad 2H_2 = R - 2H_1 = 2.2303 - 0.4912 = 1.7391 \text{ kN}$$

Force on on one bottom hinge, $$H_2 = 0.6239 \text{ kN}$$

2.9 A 9 m deep tank contains 6 m of water and 3 m of oil of relative density 0.88. Determine the pressure at the bottom of the tank. What is the pressure at the bottom of the tank if the entire tank is filled with water? What is the water thrust in this case? Draw the pressure distribution diagram in both the cases.

[8 marks : 2015]

Solution:

Case 1: Pressure at top = 0

Pressure at interface $= s_l \gamma_w z_1$

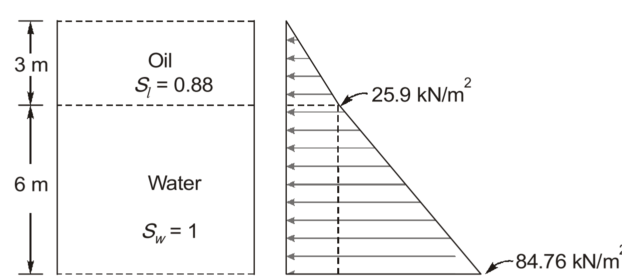
$= 0.88 \times 9.81 \times 3$

$= 25.90 \text{ kN/m}^2$

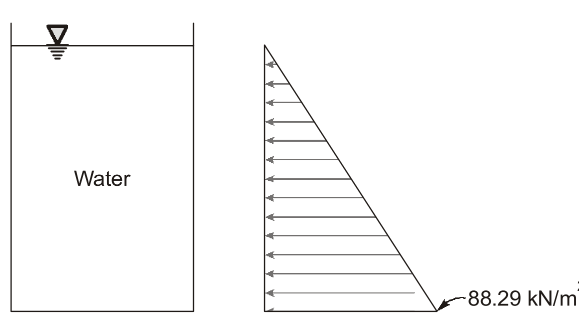
Pressure at bottom $= s_l \gamma_w z_1 + \gamma_w z_2$

$= 0.88 \times 9.81 \times 3 + 9.81 \times 6$

$= 84.76 \text{ kN/m}^2$



Case 2:



$$\text{Pressure at top} = 0$$
$$\text{Pressure at bottom} = \gamma_w H = 9 \times 9.81 = 88.29 \text{ kN/m}^2$$
$$\text{Water thrust} = \frac{1}{2}(\gamma_w H) \times H = \frac{1}{2} \times 88.29 \times 9 = 397.305 \text{ kN per meter width of wall}$$

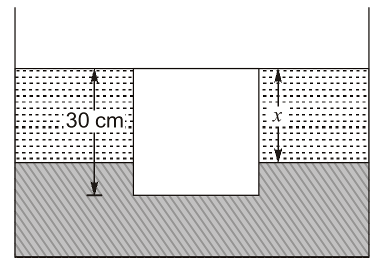
3. Buoyancy & Floatation

3.1 **A metallic cube 30 cm side and weighing 450 N is lowered into a tank containing a two fluid layer of water and mercury. Top edge of the cube is at water surface. Determine the position of block at water-mercury interface when it has reached equilibrium.**

[10 marks : 2009]

Solution:

Let the top edge of the cube be at a distance x from the water mercury interface.



As per Archimedes principle, when a body is immersed in a fluid either wholly or partially, it is lifted up by a force which is equal to the weight of the fluid displaced by the body. Thus, the force of buoyancy is given by

$$F_B = \text{Weight of water and mercury displaced by cube}$$
$$= \text{Volume of cube in water} \times \rho_w g + \text{Volume of cube in mercury} \times \rho_{Hg} g$$
$$= \left(\frac{30}{100} \times \frac{30}{100} \times x\right) \times 1000 \times 9.81 + \frac{30}{100} \times \frac{30}{100} \times \left(\frac{30}{100} - x\right) \times 13600 \times 9.81$$
$$= 0.3 \times 0.3 \times 9810 \times x + 0.3 \times 0.3 \times 13600 \times 9.81(0.3 - x)$$

$$\Rightarrow \quad F_B = 882.9x + 12007.44(0.3 - x) \quad \text{...(i)}$$

As per the principle of floatation the weight of a body floating in a fluid is equal to the buoyant force at equilibrium

$$\therefore \quad F_B = 450 \text{ N} \quad \text{...(ii)}$$

From (i) and (ii), we have

$$450 = 882.9x + 12007.44\,(0.3 - x)$$
$$\Rightarrow \quad 12007.44x - 882.9x = 3602.232 - 450$$
$$\Rightarrow \quad 11124.54x = 3152.232$$
$$\Rightarrow \quad x = \frac{3152.232}{11124.54} = 0.283 \text{ m} = 28.3 \text{ cm}$$

3.2 **A metallic sphere of specific gravity 8.0 falls in an oil of density 800 kg/m³. The diameter of the sphere is 10 mm. The viscosity of oil is 7.848 N-s/m². Determine the terminal velocity of metallic sphere.**

[4 marks : 2010]

Solution:

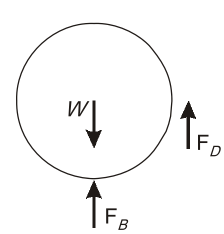
If a body drops in a fluid, at the instant it has acquired terminal velocity the net force acting on the body will be zero. The forces acting on the body at this state will be

(i) Weight of body (W) acting downward

(ii) Drag force (F_D) acting opposite to the direction of motion of body

(iii) Buoyant force (F_B) acting vertically up.

$\therefore$ Net force on the metallic sphere = $F_B + F_D - W$



But $\quad F_B + F_D - W = 0$

$$\Rightarrow \quad W = F_B + F_D \quad \text{...(i)}$$

Specific gravity of metallic sphere = 8.0

Density of metallic sphere, $\rho_s = 8 \times \rho_w = 8 \times 1000 = 8000 \text{ kg/m}^3$

Diameter of sphere, $D = 10 \text{ mm} = 10 \times 10^{-3}$ m

Viscosity of oil, $\mu = 7.848$ N-s/m^2

$\therefore \quad W = \rho_s \times g \times$ volume of sphere

$$= 8000 \times 9.81 \times \frac{\pi}{6} \times (10 \times 10^{-3})^3 = 0.0411 \text{ N}$$

and $F_B = \rho_o \times g \times$ volume of sphere

$$= 800 \times 9.81 \times \frac{\pi}{6} \times (10 \times 10^{-3})^3 = 0.00411 \text{ N}$$

Assuming Stoke's law to be valid in this case, we get

$$F_D = 3\pi\mu VD \qquad \text{where } V \text{ is the terminal velocity}$$

From (i), we get

$$0.0411 = 0.00411 + 3\pi\mu VD$$

$$\Rightarrow \quad 3\pi\mu VD = 0.0411 - 0.00411$$

$$\Rightarrow \quad V = \frac{0.03699}{3\pi \times 7.848 \times 10 \times 10^{-3}} = 0.05 \text{ m/s}$$

But Stoke's law is valid only upto Reynolds number less than 0.2

$$\therefore \quad \text{Re} = \frac{\rho_o VD}{\mu} = \frac{800 \times 0.05 \times 10 \times 10^{-3}}{7.848} = 0.051 < 0.2 \text{ (ok)}$$

Thus the terminal velocity of the sphere is 0.05 m/s

3.3 **A vertical gap 23.5 mm wide of infinite extent contains oil of specific gravity 0.9 and viscosity 2.5 N-s/m^2. A metal plate 1.5 m × 1.5 m × 1.5 mm weighing 50 N is to be lifted through the gap at a constant speed of 0.1 m/sec. Estimate the force required to lift the plate.**

[6 marks : 2012]

Solution:

Width of gap = 23.5 mm

Viscosity, μ = 2.5 Ns/m^2

Specific gravity oil = 0.9

$\therefore$ Weight density of oil = $0.9 \times 1000 = 900$ kgf/m^3

= 900×9.81 N/m^3 ($\because$ 1 kgf = 9.81 N)

Assuming that the plate lies in the middle of the gap

Volume of plate = 1.5 m × 1.5 m × 1.5 mm

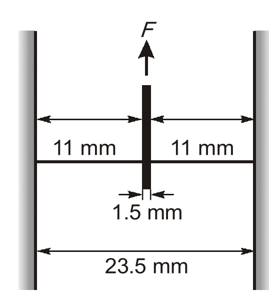
= $1.5 \times 1.5 \times 0.0015$ m^3

= 0.003375 m^3

Thickness of plate = 1.5 mm

Velocity of plate = 0.1 m/sec

Weight of plate = 50 N



When the plate is in the middle of the gap, the distance of plate from vertical surface of the gap

$$= \left(\frac{\text{Width of gap} - \text{Thickness of plate}}{2}\right)$$

$$= \left(\frac{23.5 - 1.5}{2}\right) = 11 \text{ mm} = 0.011 \text{ m}$$

Now, shear force on left side of the metallic plate

$$F_1 = \text{Shear stress} \times \text{Area}$$

$$= \mu\left(\frac{du}{dy}\right)_1 \times (1.5 \times 1.5) = 2.5 \times \left(\frac{0.1}{0.011}\right) \times 1.5 \times 1.5 = 51.136 \text{ N}$$

Similarly, the shear force on the right side of the metallic plate

$$F_2 = \text{Shear stress} \times \text{Area}$$

$$= 2.5 \times \left(\frac{0.1}{0.011}\right) \times (1.5 \times 1.5) = 51.136 \text{ N}$$

$\therefore$ Total shear force $= F_1 + F_2 = 51.136 + 51.136 = 102.272$ N

In this case the weight of plate (which is acting downward) and upward thrust is also to be taken into account.

$\therefore$ The upward thrust = weight of fluid displaced = ρvg

= (unit weight of fluid) × Volume of fluid displaced

= $9.81 \times 900 \times 0.003375$

= 29.80 N

The net force acting in the downward direction due to the weight of the plate and upward thrust

= weight of plate – upward thrust = 50 – 29.80 = 20.20 N

$\therefore$ Total force required to lift the plate up

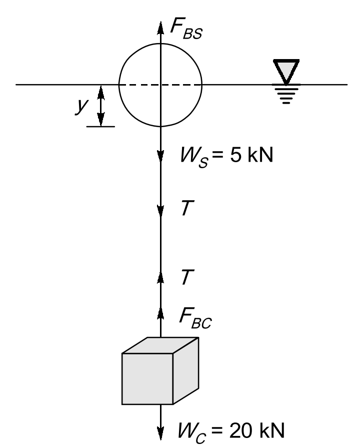
= Total shear force + 20.20 = 102.272 + 20.20 = 122.472 N

3.4 **The weights of a cube (side = 1.2 m) and a sphere (diameter = 1.25 m) are 20 kN and 5 kN respectively. Both cube and sphere are connected together by a short rope in a water reservoir. Computer the tension in the rope and percentage of sphere that will be above water surface.**

[8 marks : 2016]

Solution:

Let subscript '*s*' denotes sphere and subscript '*c*' denotes cube and T be tension in the rope.



y = Depth of immersion of sphere

Given: Diameter of sphere (d_s) = 1.25 m

Side of cube (a_c) = 1.2 m

For cube, $T = W_C - F_{BC}$

$= 20 - (a_c)^3\, 9.81$

$= 20 - (1.2)^3\, 9.81$

$\therefore$ Tension in the rope, $T = 3.048$ kN

For sphere $T + W_S = F_{BS}$

$\Rightarrow$ $3.048 \times 10^3 + 5 \times 10^3 = 1000 \times V_{immersed} \times 9.81$

$\Rightarrow$ $V_{immersed} = 0.8204 \text{ m}^3$ = Immersed volume sphere in water.

$\because$ Volume of sphere in air $= \left[\frac{\pi}{6}(1.25)^3 - 0.8204\right] \text{m}^3 = 0.2023 \text{ m}^3$

$\therefore$ Percentage of sphere above water surface $= \dfrac{0.2023}{\frac{\pi}{6}(1.25)^3} \times 100 = 19.78\%$