$\boxed{\text{C. T. F. S.}}$ → Part - 2

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Comprehensive Course on Signal System

Continuous Time ~~Fourier~~ Transform - Part I

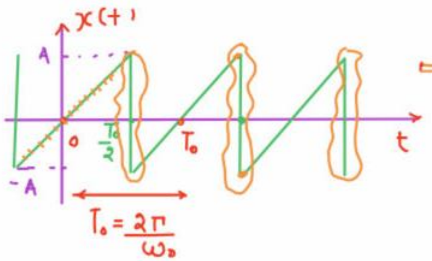
Lesson 32 • June 24, 2020 • Vishal Soni

$\boxed{\text{RECAP:}}$

Any vector can be written as weighted sum of mutually orthogonal unit vectors.

↓

Any T.D. nonsinusoidal periodic signal with $T_0 = \frac{2\pi}{\omega_0}$ canbe written as weighted sum of mutually orthogonal signal in $(0, \frac{2\pi}{\omega_0})$.



⟹ Time domain nonsinusoidal periodic signal with $T_0 = \frac{2\pi}{\omega_0}$.

By observing $x(t)$ ⟶ Frequency can't be calculated

$x(t) \equiv x_{FS}(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \cdots$
$+ b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \cdots$

$$\boxed{x(t) \equiv x_{FS}(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t} \text{ F.S.}$$

FOURIER SERIES:

→ Fourier series is the representation of time domain nonsinusoidal periodic signal as the weighted sum of mutually orthogonal Harmonically related sinusoids.

→ Fourie series provides the method to break time domain non sinusoidal periodic signal as the weighted sum of mutually orthogonal Harmonically related sinusoids.

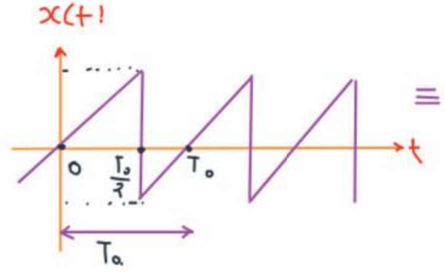
purpose: To calculate the frequency content in time domain non sinusoidal periodic signal.

→ One cannot Tell Frequency of Time domain signal by looking at time domain signal.

→ $$\boxed{x(t) \equiv x_{FS}(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)}$$

Frequency calculation.

TRIGONOMETRIC FOURIER SERIES:



$$\equiv x_{FS}(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

→ periodic with T_0

→ T.F.S. of T.D. N.S.P signal

→ a_0, a_n, b_n → TFS Coefficient

Calculation of a_0: $T_0 = 2\pi/\omega_0$

$$x(t) = a_0 + a_1 \cos\omega_0 t + \cdots + b_1 \sin\omega_0 t + \cdots$$

$$\int_0^{T_0} x(t)\,dt = \int_0^{T_0} a_0\,dt + \int_0^{T_0} a_1 \cos\omega_0 t\,dt + \cdots + \int_0^{T_0} b_1 \sin\omega_0 t\,dt + \cdots$$

$\to 0$ $\to 0$

$$\boxed{\int_0^{T_0} x(t)\,dt = a_0 T_0}$$

$$\boxed{a_0 = \frac{1}{T_0} \int_0^{T_0} x(t)\,dt}$$

$$\boxed{a_0 = \frac{1}{T_0} \int_{T_0} x(t)\,dt} \to \text{Constant}$$

$$\boxed{a_0 = \frac{\text{Area of } x(t) \text{ in } 1\,T_0}{T_0}}$$

Calculation of a_n:

$T_0 = 2\pi/\omega_0$

$$x(t) = a_0 + a_1\cos\omega_0 t + \cdots + a_n\cos n\omega_0 t + \cdots$$
$$+ b_1\sin\omega_0 t + \cdots + b_n\sin n\omega_0 t + \cdots$$

TD sig

$$\int_0^{T_0} x(t)\cos n\omega_0 t\,dt = \int_0^{T_0} a_0\cos n\omega_0 t\,dt \xrightarrow{} 0 + \int_0^{T_0} a_1\cos\omega_0 t\cos n\omega_0 t\,dt \xrightarrow{} 0$$
$$+ \int_0^{T_0} a_n\cos^2 n\omega_0 t\,dt + \cdots \quad \leftarrow \frac{a_n \cdot T_0}{2}$$
$$+ \int b_1\sin\omega_0 t\cos n\omega_0 t\,dt \,(\to 0) + \cdots + \int b_n\sin n\omega_0 t\cos n\omega_0 t\,dt \,(\to 0)$$

$$\int_0^{T_0} x(t)\cos n\omega_0 t\,dt = \frac{a_n T_0}{2}$$

$$\boxed{a_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cos n\omega_0 t\,dt}$$

Calculation of $b_n \longrightarrow$ do it yourself.

TFS coefficients:

$$\boxed{a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt} \qquad T_0 = \frac{2\pi}{\omega_0}$$

$$\boxed{a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega_0 t\,dt} \qquad n \geq 1$$

$$\boxed{b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega_0 t\,dt} \qquad n \geq 1$$

Note: TFS can be written if $x(t)$ is

- → Real
- → complex
- → purely Imaginary.

$x(t)$	a_0	a_n	b_n
Real →	R	R	R
Complex →	C	C	C
Purely Imaginary →	I	I	I

$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt \longrightarrow$ constant

$a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega_0 t\,dt = f(n\omega_0)$ $\boxed{n \geq 1}$

$\boxed{a_{-n} = a_n}$

$b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega_0 t\,dt = g(n\omega_0)$ $\boxed{n \geq 1}$

$\boxed{b_{-n} = -b_n}$

POLAR FORM OF T.F.S.

→ valid only when $x(t)$ is Real T.D. nonsinusoidal periodic signal.

→ $x(t)$ is Real → a_0, a_n, b_n → $\boxed{\text{REAL}}$

$$a_n = \begin{cases} a_n & n \geq 1 \\ a_0 & n = 0 \end{cases} \qquad b_n = \begin{cases} b_n & n \geq 1 \\ b_0 = 0 & n = 0 \end{cases}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$\begin{cases} a_n = r_n \cos\phi_n \\ b_n = r_n \sin\phi_n \end{cases}$$

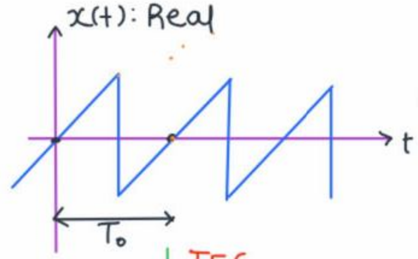
$r_n = \sqrt{a_n^2 + b_n^2} = \begin{cases} r_n = \sqrt{a_n^2 + b_n^2} & n \geq 1 \\ r_0 = a_0 & n = 0 \end{cases}$

$\phi_n = \tan^{-1}\frac{b_n}{a_n} = \begin{cases} \phi_n = \tan^{-1}\frac{b_n}{a_n} & n \geq 1 \\ \phi_0 = 0° & n = 0 \end{cases}$

$$x(t) = a_0 + \sum_{n=1}^{\infty} (r_n \cos\phi_n \cos n\omega_0 t + r_n \sin\phi_n \sin n\omega_0 t)$$

$$\boxed{x(t) = r_0 + \sum_{n=1}^{\infty} r_n \cos(n\omega_0 t - \phi_n)}$$

POLAR FORM OF T.F.S.



$$\equiv x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

POLAR

$$x(t) = r_0 + \sum_{n=1}^{\infty} (r_n \cos(n\omega_0 t - \phi_n))$$

TFS → POLAR

$$x(t) = \underbrace{r_0} + \underbrace{r_1 \cos(\omega_0 t - \phi_1)} + \underbrace{r_2 \cos(2\omega_0 t - \phi_2)} + \cdots\cdots$$

NOTE Sinusoidal periodic signal → single discrete freq.

nonsinusoidal periodic signal → Infinite no of discrete freq. which are integer multiples of ω_0.

\# r_0 → dc component of Timedomain nonsinusoidal periodic signal $x(t)$.

- → Freq of dc component = 0 Hz.
- → amplitude : r_0
- → power or MSV = r_0^2
- → rms value = r_0

$r_1 \cos(\omega_0 t - \phi_1)$

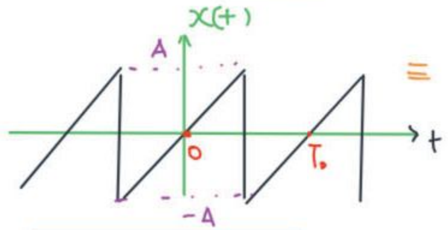
- I^{st} or Fundamental Harmonic of Timedomain non sinusoidal periodic signal $x(t)$.
- Freq^n of I Harmonic $\begin{cases} \omega_0 \\ f_0 \end{cases}$
- amplitude: $r_1 = \sqrt{a_1^2 + b_1^2}$
- rms value: $\frac{r_1}{\sqrt{2}} = \frac{\sqrt{a_1^2 + b_1^2}}{\sqrt{2}}$
- MSV or power $= \frac{r_1^2}{2} = \frac{a_1^2 + b_1^2}{2}$

<u>Note</u>: For any Time domain non sinusoidal periodic signal K^{th} Harmonic will be:

$r_K \cos(K\omega_0 t - \phi_K)$

- Freq: $K\omega_0$
- Amp: $r_K = \sqrt{a_K^2 + b_K^2}$
- rms: $\frac{r_K}{\sqrt{2}}$
- power: $\frac{r_K^2}{2}$

Power Calculation:



$\equiv \quad x_{FS}(t) = r_0 + r_1 \cos(\omega_0 t - \phi_1) + r_2 \cos(2\omega_0 t - \phi_2) + r_3 \cos(3\omega_0 t - \phi_3) + \ldots\ldots$

$\boxed{P_x = \overline{x^2(t)} = \frac{A^2}{3}} \equiv P_x = r_0^2 + \frac{r_1^2}{2} + \frac{r_2^2}{2} + \frac{r_3^2}{2} + \ldots\ldots$

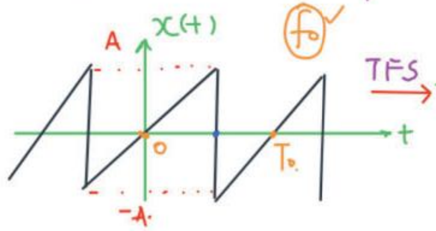
$P_x = \overline{x^2(t)} = \text{MSV}\{x(t)\} = r_0^2 + \frac{r_1^2}{2} + \frac{r_2^2}{2} + \frac{r_3^2}{2} + \ldots\ldots$

$\text{RMS}\{x(t)\} = \sqrt{r_0^2 + \frac{r_1^2}{2} + \frac{r_2^2}{2} + \frac{r_3^2}{2} + \ldots}$

$\boxed{P_{x_{FS}} = \text{m\% of } x(t)}$

power in FS = m% of TD $x(t)$

Frequency domain Analysis of Time domain non sinusoidal periodic signal:

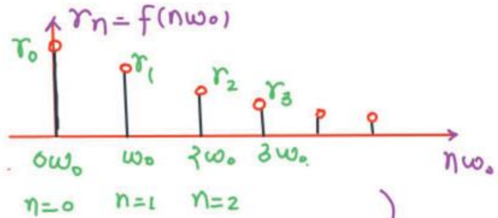


f_0

$$x(t) \xrightarrow{TFS} x(t) = r_0 + r_1 \cos(\omega_0 t - \phi_1) + r_2 \cos(2\omega_0 t - \phi_2) + r_3 \cos(3\omega_0 t - \phi_3) + r_4 \cos(4\omega_0 t - \phi_4) + \cdots$$

$r_n = ?$ → $r_1 = 0$, $r_3 = 0$, $r_5 = 0$

$$x(t) \xrightarrow{F.D.}$$



Spectrum.

In Time domain nonsinusoidal periodic signal those Harmonics will be absent For which

$$\boxed{r_n = 0}$$

$$\boxed{r_K = 0 \longrightarrow K^{th} \text{ Harmonic will be absent}}$$

$$x(t) \xleftrightarrow{F.D.} r_n = f(n\omega_0) = \sqrt{a_n^2 + b_n^2}$$

$$= \begin{cases} \sqrt{a_n^2 + b_n^2} & n \geq 1 \\ a_0 & n = 0 \end{cases}$$

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Continuous Time Fourier Transform - Part II

Lesson 33 • June 25, 2020 • Vishal Soni

RECAP:

T.F.S.

$$\boxed{x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)}$$

x(t) is Real, complex, purely Im.

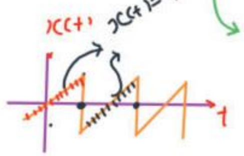
If x(t): Real

$$x(t) = r_0 + \sum_{n=1}^{\infty} (r_n \cos(n\omega_0 t - \phi_n))$$

polar form of T.F.S.

T.F.S.:

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$



$$a_0 = \frac{1}{T_0} \int_{T_0} x(t)\,dt \checkmark$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t\,dt \checkmark$$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t\,dt \checkmark$$

Complete set of mutually orthogonal, Harmonically related complex exponentials in $(0, \frac{2\pi}{\omega_0}) \rightarrow T_0 = \frac{2\pi}{\omega_0}$

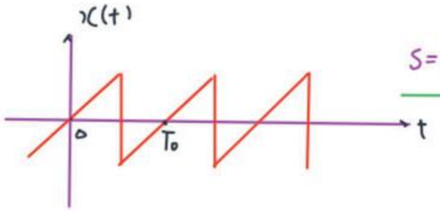
S_1: $\{e^{jn\omega_0 t}\}_{n=-\infty}^{\infty}$ in the interval $(0, \frac{2\pi}{\omega_0})$

$S = \{ \ldots\ldots e^{-j\omega_0 t}, 1, e^{j\omega_0 t}, \ldots\ldots \}$ in $(0, \frac{2\pi}{\omega_0})$

Inner product:

$$\int_0^{2\pi/\omega_0} e^{+j\omega_0 t}(e^{-j\omega_0 t})^* dt = \int_0^{2\pi/\omega_0} e^{j2\omega_0 t} dt = 0$$

$$\int_0^{2\pi/\omega_0} e^{j\omega_0 t}(e^{j\omega_0 t})^* dt = \int_0^{2\pi/\omega_0} 1 \, dt = \frac{2\pi}{\omega_0} \neq 0$$

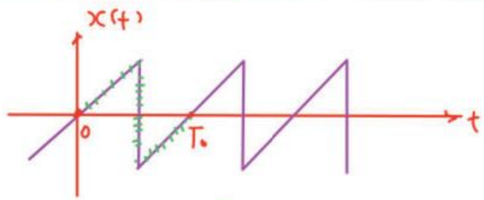


$S = \{e^{jn\omega_0 t}\}_{n=-\infty}^{\infty}$

$$x(t) = \cdots + C_{-1}e^{-j\omega_0 t} + C_0 e^{j0\omega_0 t} + C_1 e^{j\omega_0 t} + C_2 e^{j(2\omega_0)t} + \cdots$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}} \rightarrow$$ complex or exponential Fourier series.

Complex or Exponential Fourier Series:



$\equiv$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}}$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

**

$$\boxed{\begin{aligned} C_0 &= \frac{1}{T_0}\int_{T_0} x(t)\,dt = a_0 = r_0 \\ C_n &= \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}\,dt \end{aligned}}$$

$-\infty < n < \infty$

$$\boxed{C_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}\,dt}$$

$$C_n = \frac{1}{T_0}\int_{T_0} x(t)\cos n\omega_0 t\,dt - j\frac{1}{T_0}\int_{T_0} x(t)\sin n\omega_0 t\,dt$$

$$\boxed{C_n = \frac{a_n}{2} - j\frac{b_n}{2}}$$

$$C_n = \frac{a_n}{2} - j\frac{b_n}{2}$$

$-\infty < n < \infty$

$$\boxed{\begin{aligned} a_n &= f(n\omega_0) \quad n \geq 1 \\ b_n &= g(n\omega_0) \quad n \geq 1 \end{aligned}}$$

$$\boxed{\begin{aligned} C_n &= \frac{a_n}{2} - j\frac{b_n}{2} \quad & n \neq 0 \\ C_0 &= a_0 = r_0 \quad & n = 0 \end{aligned}}$$

Summary: 1) $x(t) \overset{TFS}{=} a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$

$$\boxed{\begin{aligned} a_0 &= \frac{1}{T_0}\int_{T_0} x(t)\,dt \\ a_n &= \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega_0 t\,dt \quad & n \geq 1 \\ b_n &= \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega_0 t\,dt \quad & n \geq 1 \end{aligned}}$$

TFS is applied $x(t) \longrightarrow$ Complex, Real or J