Strength of Materials

1. Properties of material & axial stresses
2. Bending moment & shear force diagram
3. Deflection
4. Transformation of stresses and strains
5. Combined stress

(*****)
(objective & conventional)

6. Bending stress
7. Shear stress
8. Torsion

(***)
(objective & conventional)

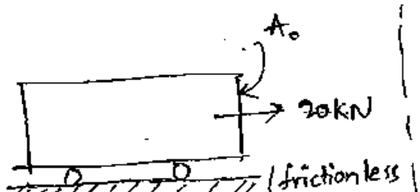
9. Thick & Thin shell
10. Springs
11. Columns
12. Moment of Inertia

(*)
(objective)

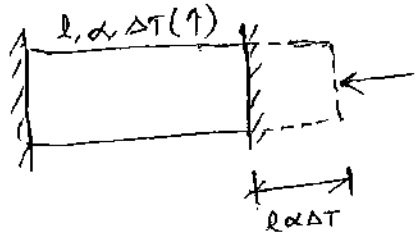
- Properties of Materials & Axial stresses

→ A stress developed in a body on account of restraining force and restrained deformation.

→



∴ $\boxed{\text{Stress}=0}$
(because no resisting force)



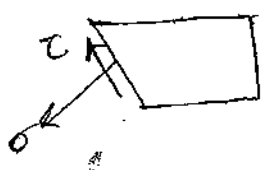
$\Rightarrow \frac{Rl}{AE} = l\alpha\Delta T$

$\Rightarrow \sigma = \frac{R}{A} = E\alpha T$

→ stresses are of 2 types.

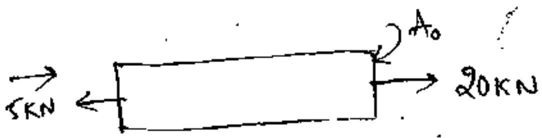
A) Normal stress (⊥lar to surface)

B) Shear stress (Along the surface)



σ = Normal stress

τ = shear stress



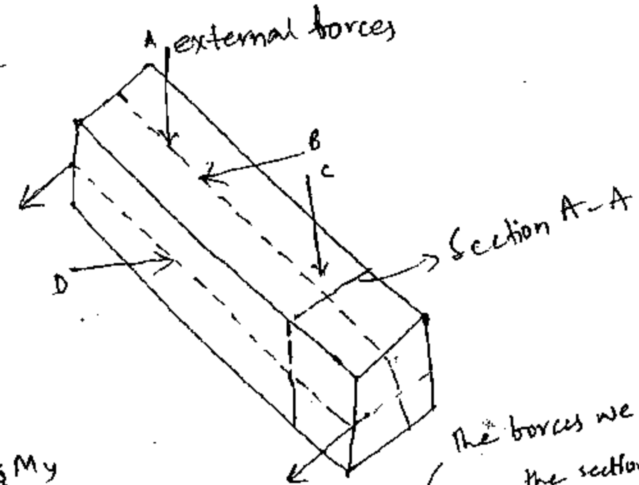
$$\sigma = \frac{\text{Resisting force}}{\text{Area}}$$

Here, $\boxed{\sigma = \frac{5}{A_0}}$

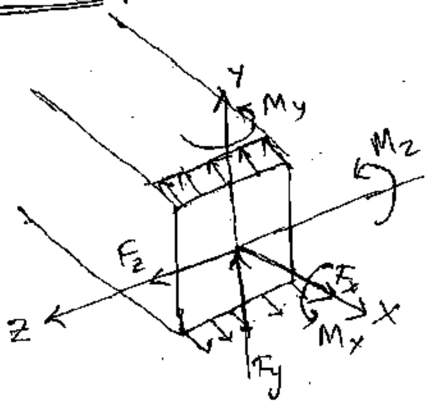
→ Internal force & External forces :-

→ The forces we can see : "External forces". (A,B,C,D).

3D-Case :-



Section A-A :-



The forces we can see once we cut the section are "Internal forces".

Internal forces :- F_x, F_y, F_z
M_x, M_y, M_z

→ Direction of moments is given by "Right Hand Thumb rule".

→ When the structure and loading are not in same plane, the condition is called "3D-Condition".

→ However if loading as well as structure are in same plane, the condition is called "2D-Condition".

→ In 3D-Conditions no. of internal forces is six ($F_x, F_y, F_z, M_x, M_y, M_z$).

i.e, F_x = axial force $\xrightarrow[\text{develop}]{\text{will}}$ Normal stress [axial stress]

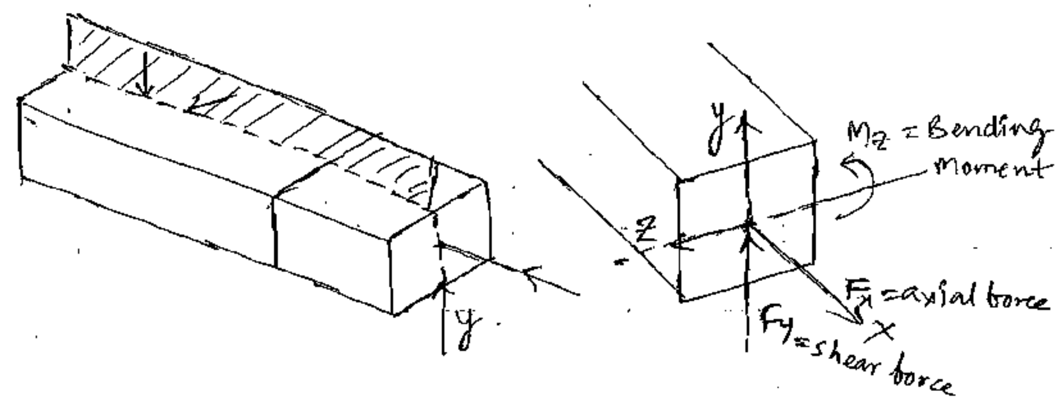
F_y = } Shear force ⟶ shear stress [Transverse shear stress]
F_z =

M_x = Twisting moment ⟶ shear stress [Torsional shear stress]

M_y = } Bending moment ⟶ Normal stress [Bending stress]
M_z =

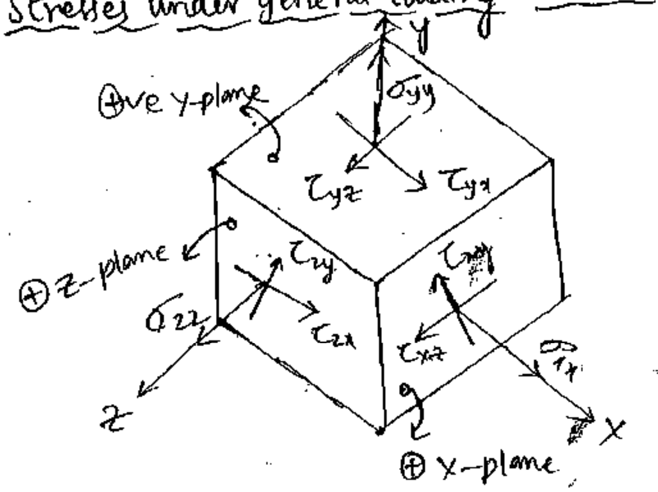
→ <u>2D-Case</u> ;

$F_z(x)$
$M_x(x)$
$M_y(x)$



→ In case of 2D structure no. of internal forces is 3. $[F_x, F_y, M_z]$
i.e, Axial force, shear force, Bending moment.

→ <u>Stresses under general loading condition</u> :-



<u>Naming</u> ;

Normal stress → $\sigma_{\square\square}$

Shear stress → $\tau_{\square\square}$

(plane on which stress acts) (direction in which the stress acts)

⊕ x-plane → Outward normal to plane is in +ve x-direction

⊕ y-Plane →

⊕ z-plane →

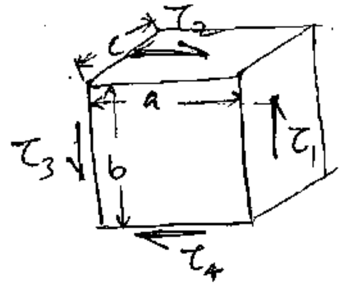
⊖ x-Plane → Outward normal to plane is in -ve x-direction.

⊖ y-Plane →

⊖ z-plane →

→ At any point inside the body under general loading condition, no. of stresses is $\underline{9}$. $[\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{xz}, \tau_{yx}, \tau_{yz}, \tau_{zy}, \tau_{zx}]$.

→ $\left[\begin{array}{l} 3 - \text{Normal stresses} \\ 6 - \text{Shear stress} \end{array}\right\} = 9.\Big]$



σ_{xx} | $\tau_{xy} = \tau_{yx}$

σ_{yy} | $\tau_{yz} = \tau_{zy}$

σ_{zz} | $\tau_{zx} = \tau_{xz}$

i) $\Sigma F_V = 0$

→ $\tau_1 bc - \tau_3 bc = 0$

⇒ $\boxed{\tau_1 = \tau_3}$

ii) $\Sigma F_H = 0$

→ $\tau_2 ac - \tau_4 ac = 0$.

⇒ $\boxed{\tau_2 = \tau_4}$

iii) $\Sigma M_{\circlearrowright} = 0$.

→ $(\tau_2 ac) \times b - (\tau_1 bc) a = 0$.

⇒ $\boxed{\tau_1 = \tau_2}$

→ Shear stresses on opposite faces are equal in magnitude and opposite in direction. (followed from <u>Force equilibrium</u>).

→ Shear stresses on adjacent perpendicular planes are equal in magnitude and are directed either towards each other or away from each other. (as followed from <u>Moment equilibrium</u>).

→ Hence, $\tau_{xy} = \tau_{yx}$

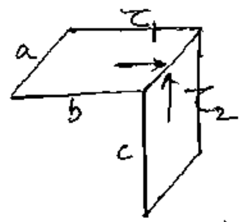
$\tau_{yz} = \tau_{zy}$

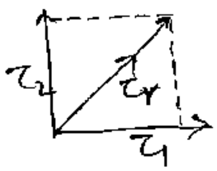
$\tau_{zx} = \tau_{xz}$

→ At any point inside the body under general loading condition, no. of independent stress components is $\underline{6}$. $[\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}]$.

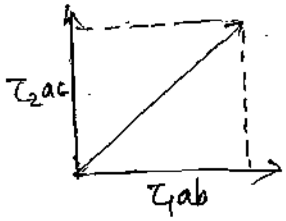
→ Stress is "<u>not a vector</u>". It has direction as well as magnitude but resultant of 2 stresses can not be obtained using "<u>Parallelogram law of vector addition</u>".

ex:-





$\tau_r \neq \sqrt{\tau_1^2+\tau_2^2}$



, $R = \sqrt{(\tau_2 ac)^2 + (\tau_1 ab)^2}$

→ Stress is named mathematical quantity as "<u>Tensor</u>".

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

∴ Stress Tensor matrix

→ Principle diagonal.

→ Stress Tensor matrix is a "<u>Symmetrical matrix</u>".

→ Stress Tensor matrix is a "Symmetrical matrix". & Symmetry of ~~Stress~~ Tensor matrix is on account of "<u>Moment equilibrium</u>".

→ Stress is a "<u>Second order</u>" Tensor.

→ Strain is also a "<u>Second order</u>" Tensor

No. of elements = 3^n

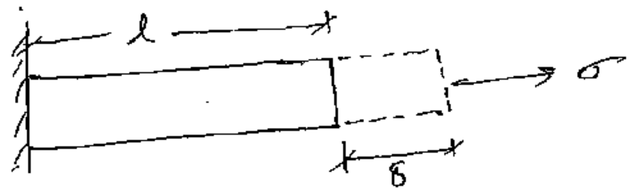
(n = order of Tensor)

<u>Note</u>:-

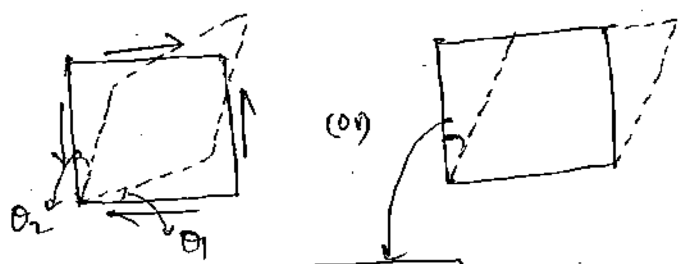
→ Strain and moment of Inertia are also "<u>II^{nd} Order</u>" Tensor

→ Under plane stress condition no. of stress components are σ_x, σ_y, τ_{xy}, τ_{yx} but $\tau_{xy} = \tau_{yx}$, So independent stress components are <u>3</u> [i.e, σ_x, σ_y, τ_{xy}]

⇒ Strain :-

→ 

$$\boxed{\frac{\delta}{l} = \varepsilon = \text{Normal strain}}$$

→ 

$$\boxed{\theta_1 + \theta_2 = \gamma = \text{Shearing strain}}$$

→ "Strain" is a fundamental quantity not the stress, because strains can be measured using strain gauge but stress only be derived as

$$= \frac{\text{Resisting force}}{\text{Area}}$$

→ "Only Normal strains" can be measured not shearing strains.

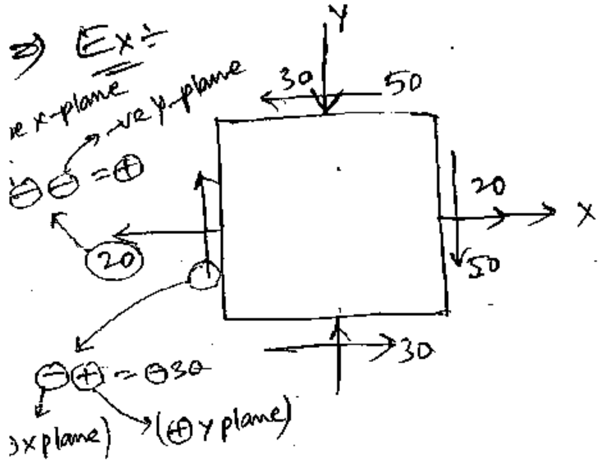
→ Sign convention for stresses :-

→ $\sigma_{\square\square}$

τ_{pa} → direction; p → Plane

Plane	direction		Sign
⊕	⊕	=	⊕
⊕	⊖	=	⊖
⊖	⊕	=	⊖
⊖	⊖	=	⊕

⇒ Ex :-



$\sigma_x = \oplus 20$ (Tension)

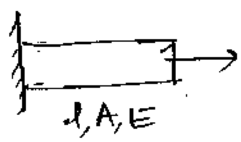
$\sigma_y = \ominus 30$ (Compression)

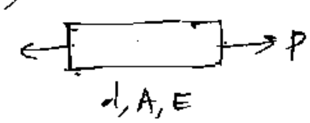
$\tau_{xy} = \ominus 50$

⇒ Stress strain curve :-

Note :-

- Rigid body traslation has no effect on "stresses"
(or) motion

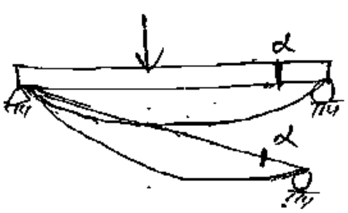
i) 

ii) 

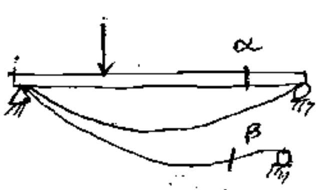
(Both (i) & (ii) have same stress)

→ If the motion leads to deformation in the body, it is not a rigid body motion.

→ If the motion leads to no deformation in the body, this motion is called "Rigid body motion".



[∵ Rigid body motion]
→ ($\alpha = \alpha$)



[∵ Not Rigid body motion]
→ ($\alpha \neq \beta$)