# MADE EASY

## Civil Engineering

### Toppers Handwritten Notes

### Hydrolics Mechanics

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# Hydraulic Machines



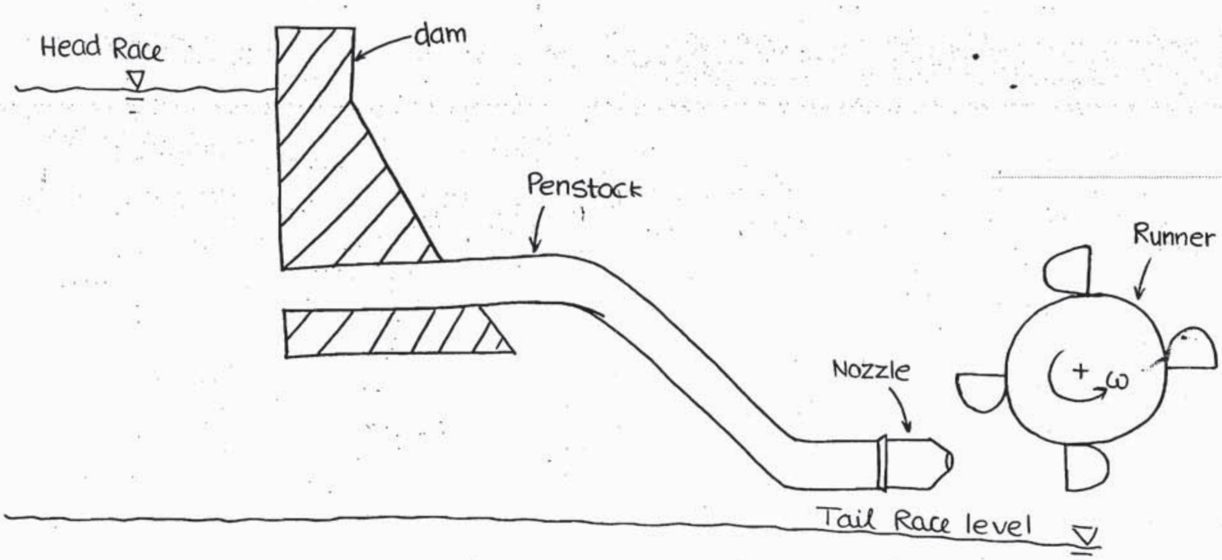
Flow in motion ⟶ Fluid dynamics

Classification of hydraulic Machines -

Work is done by the water (Ex- turbine)

Work is done on the water (Ex- Pump)

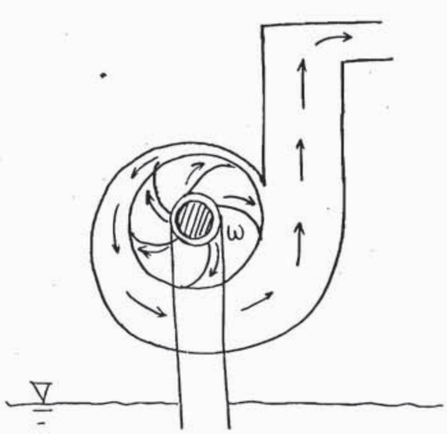
Layout of hydro electric Power plant -




Hydraulic Energy ⟶ Mechanical shaft power output ⟶ Electric Power Output

$\underbrace{\text{Hydraulic Energy} \longrightarrow \text{Mechanical shaft power output}}_{\text{Turbine}}$

Pump:-




Electric Energy ⟶ Mechanical shaft power input ⟶ Hydraulic Energy

$\underbrace{\text{Mechanical shaft power input} \longrightarrow \text{Hydraulic Energy}}_{\text{Pump}}$

In General-

Energy (Joule)

$= mgH$

$= \frac{1}{2} m V^2$

$= F \times (X)$ (Workdone)

Power (Joule/sec)

$= \dot{m} g H$

$= \frac{1}{2} \dot{m} V^2$

$= F \times u \left( \frac{\text{Workdone}}{\text{sec}} \right)$

$= T \times \omega$

Work done per sec per unit weight of water striking per sec

$= \frac{\text{Work done per sec}}{\dot{m} g} = H \ (m)$

In General-

Water

(System)

A/C to Newton's 2nd law of motion

$$\vec{F} = \underbrace{\dot{m} \bar{V}_2}_{\substack{\text{Momentum of} \\ \text{water leaving per} \\ \text{sec.}}} - \underbrace{\dot{m} \bar{V}_1}_{\substack{\text{Momentum of} \\ \text{water entering} \\ \text{per sec.}}}$$

$\dot{m}$ = mass flow rate actually striking over vane.

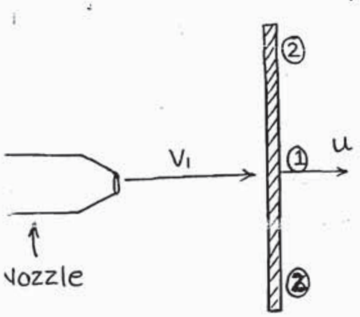
$\vec{F}$ = Force applied on the water by the vane.

Force applied by the water on the vane $= -\vec{F}$

$= \dot{m} \vec{V_1} - \dot{m} \vec{V_2}$

In General-

Notations :-




inlet of vane ⇒ 1

exit of vane ⇒ 2

$\vec{V_1}$ = absolute velocity of water at the inlet of vane

$\vec{u_1}$ = absolute " " vane " " " " "

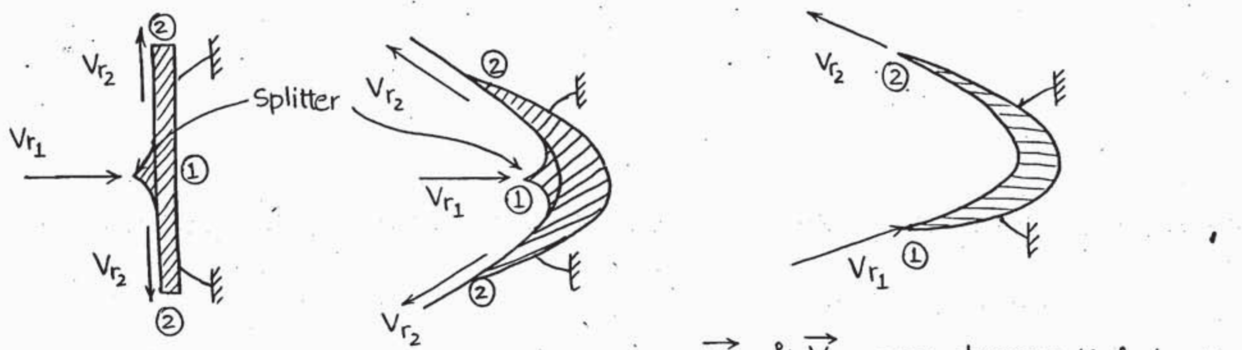
$\vec{V_{r_1}}$ = Relative " " water " " " " "

$\vec{V_2}$ = Absolute velocity of water at the exit of vane.

$\vec{u_2}$ = Absolute " " vane " " " " "

$\vec{V_{r_2}}$ = Relative " " water " " " " "

No Shock condition :- 




$\vec{V_{r_1}}$ & $\vec{V_{r_2}}$ are tangential to the vane geometry at inlet & exit respectively.

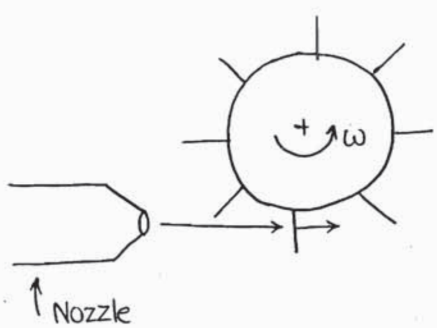
Impact of jet :-

In General -

Level-1 :- Stationary surface :-

Level-2 :- Moving surface :-

Level-3 :- impact of jet on series of flat plates -

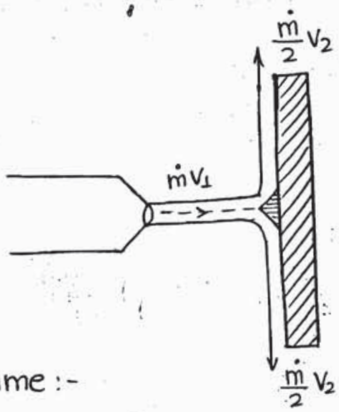



Aim -
$F_x \Rightarrow Max.$
$F_y \Rightarrow 0$

Level-1 :- Stationary Plate :-

Assume smooth blade & if there is no energy loss in the flow because of impact of the fluid jet & the difference in elevation b/w incoming & outgoing jet is neglected, the bernaulli $eq^n$ indicates that the jet will move on & off the plate with same velocity.

1) Jet strikes normally :-




d = dia. of jet.

a = c/s area of jet.

$$\left[\dot{m} = \rho a V_1\right]$$

in x-direction

$F_x = \dot{m}V_1 - 0$

$$\left[F_x = \rho a V_1^2\right]$$

Resultant

in Y-direction

$$F_y = 0 - \left[\left(\frac{\dot{m}}{2}V_2\right) + \left(-\frac{\dot{m}}{2}V_2\right)\right]$$

$F_y = 0$

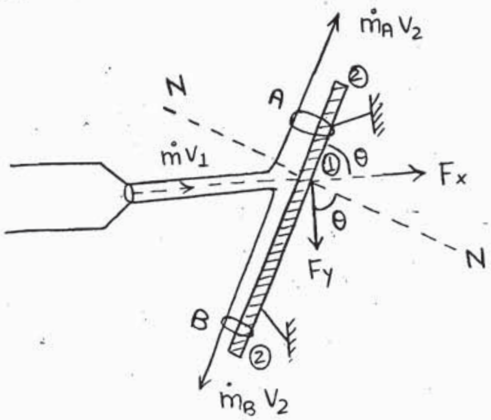
Assume :-

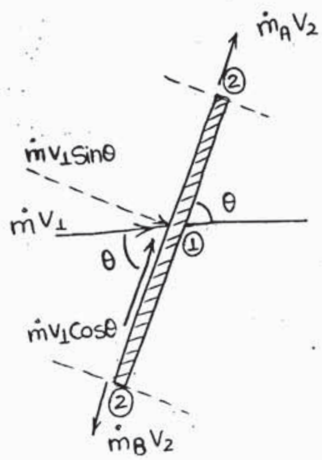
Smooth surface

$V_2 = V_1$

Note :- If the jet strikes the flat plate, then the resultant force always acts in the normal direction to the plate.

2) Jet strikes on inclined plane :-




(Assume - smooth surface - $V_2 = V_1$)

$$\dot{m} = \rho a V_1$$

in N-N direction -

$$F_N = \dot{m}V_1 \sin\theta - 0$$

$$\left[F_N = \rho a V_1^2 \sin\theta\right] \text{(Resultant)}$$

in x-direction -

$$F_x = F_N \sin\theta$$

$$\left[F_x = \rho a V_1^2 \sin^2\theta\right]$$

in y-direction -

$$F_y = F_N \cos\theta$$

$$\left[F_y = \rho a V_1^2 \sin\theta \cos\theta\right]$$

Note :- There is no force acting in the tangential direction to the plate. So, $F_N$ is the resultant dynamic thrust exerted by the jet

Force along the plate (tangential direction)

$$\dot{m} V_1 \cos\theta - \left[\dot{m}_A V_2 + (-\dot{m}_B V_2)\right] = 0$$

$$\rho Q V \cos\theta - (\rho Q_A V - \rho Q_B V) = 0$$

$$Q\cos\theta = Q_A - Q_B \quad \text{--- (1)}$$

$$\left\{\begin{aligned} \dot{m} &= \rho Q \\ \dot{m}_A &= \rho Q_A \\ \dot{m}_B &= \rho Q_B \end{aligned}\right.$$

Applying continuity -

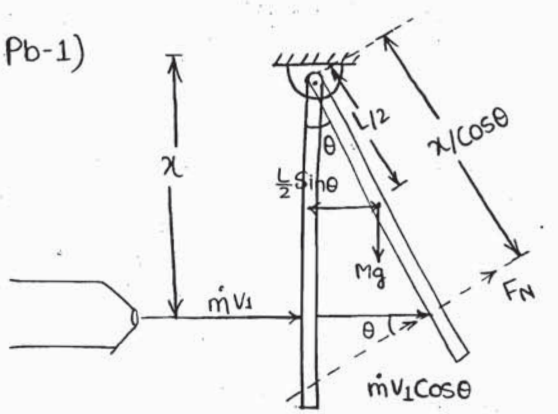
$$\dot{m} = \dot{m}_A + \dot{m}_B$$

$$Q = Q_A + Q_B \quad \text{--- (2)}$$

By eq-(1) & (2)

$$\left\{\begin{aligned} Q_A &= \frac{Q}{2}\left[1 + \cos\theta\right] \\ Q_B &= \frac{Q}{2}\left[1 - \cos\theta\right] \end{aligned}\right\}$$

Pb-1)




Taking Moments about hinge O -

$$Mg \times \frac{L}{2} \sin\theta = F_N \times \frac{x}{\cos\theta}$$

$$F_N = \dot{m} V_1 \cos\theta - 0$$

$$F_N = \rho a V_1^2 \cos\theta$$

$$Mg \times \frac{L}{2} \sin\theta = \rho a V_1^2 \cos\theta \times \frac{x}{\cos\theta}$$

$$\theta = \sin^{-1}\left[\frac{2 \rho a V_1^2 x}{Mg L}\right]$$

Given -

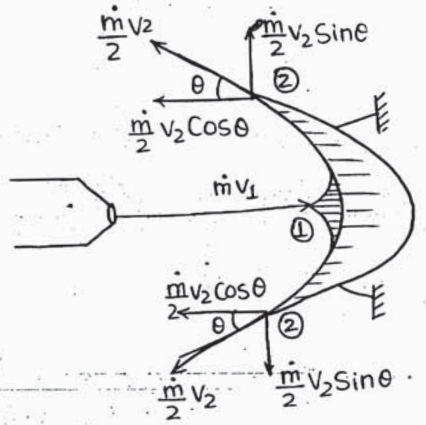
wt. of Plate = Mg

Length of plate = L

3) Curved Surface ( Symmetrical)

(Attack at centre)

$\dot{m} = \rho a V_1$




In x-direction-

$$F_x = \dot m V_1 - \left[-\frac{\dot m}{2}V_2\cos\theta + \left(-\frac{\dot m}{2}V_2\cos\theta\right)\right]$$

$$F_x = \dot m V_1(1+\cos\theta)$$

$$\left[F_x = \rho a V_1^2 \cancel{\cos}(1+\cos\theta)\right]$$

In Y-direction-

$$F_y = 0 - \left[\frac{\dot m}{2}V_2\cancel{\sin\theta} + \left(-\frac{\dot m}{2}\cancel{V_2}\sin\theta\right)\right]$$

$$\left[F_y = 0\right]$$

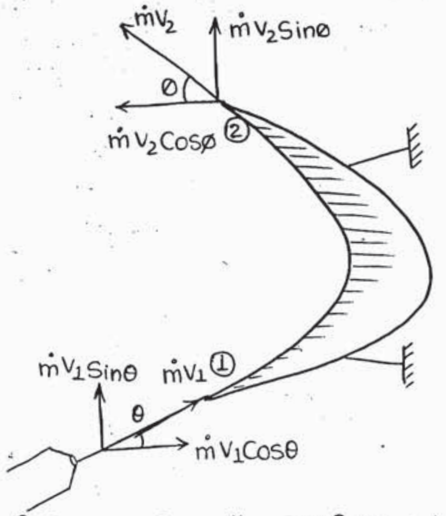
assume - Smooth surface

$V_2 = V_1$

4) Curved surface (unsymmetrical) :-

(Attack at tip)

$\dot m = \rho a V_1$




In x-direction :-

$$F_x = \dot m V_1\cos\theta - [-\dot m V_2\cos\phi]$$

$$\left[F_x = \dot m V_1(\cos\theta + \cos\phi) = \rho a V_1^2(\cos\theta + \cos\phi)\right]$$

In Y-direction :-

$$F_y = \dot m V_1\sin\theta - \dot m V_2\sin\phi$$

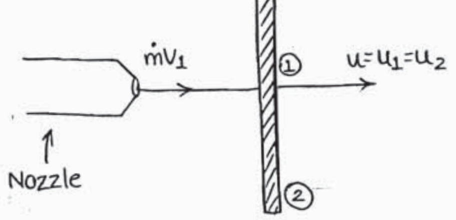
$$F_y = \dot m V_1(\sin\theta - \sin\phi)$$

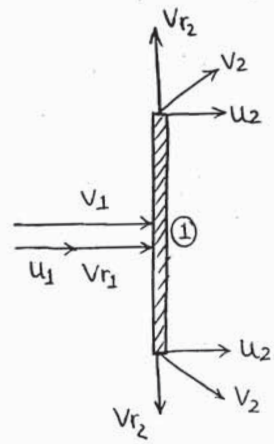
$$\left[F_y = \rho a V_1^2(\sin\theta - \sin\phi)\right]$$

Assume - Smooth Surface - $V_2 = V_1$

Level-2 :- Moving Surface :-

1) Jet Strikes normally :-




Assume -

Smooth surface -

$(V_{r_2} = V_{r_1})$

$\dot{m} = \rho a V_{r_1}$ 

In x-direction:-

$$F_x = \dot{m}V_1 - \left(\frac{\dot{m}}{2}u_2 + \frac{\dot{m}}{2}u_2\right)$$

$$(u_2 = u_1)$$

$$= \dot{m}(V_1 - u_1)$$

$$= \rho a V_{r_1}(V_1 - u_1)$$

$$V_{r_1} = (V_1 - u_1)$$

$$\left[F_x = \rho a (V_1 - u_1)^2\right]$$

In y-direction:-

$$F_y = 0 - \left[\frac{\dot{m}}{2}V_{r_2} + \left(-\frac{\dot{m}}{2}V_{r_2}\right)\right]$$

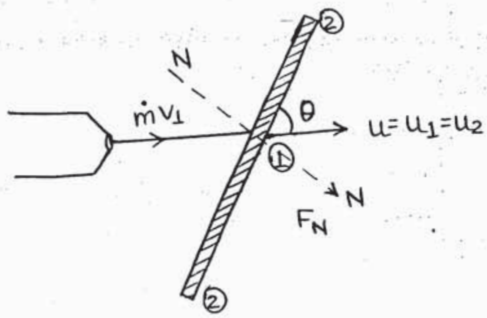
$$F_y = 0$$

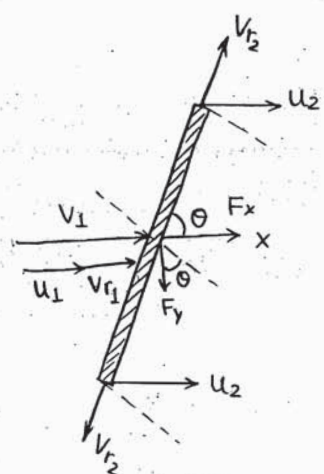
- work done per sec by the jet on the plate

$$= F_x \times u_1$$

$$= \rho a (V_1 - u_1)^2 u_1$$

2) Jet strikes at inclined plate:-




$\dot{m} = \rho a V_{r_1}$

Assume - Smooth surface

$V_{r_2} = V_{r_1}$

In N-N direction:-

$$F_N = \dot{m}V_1 \sin\theta - [\dot{m}u_2 \sin\theta]$$

$$(u_2 = u_1)$$

$$= \dot{m}(V_1 - u_1)\sin\theta$$

$$= \rho a V_{r_1}(V_1 - u_1)\sin\theta$$

$$(V_{r_2} = V_1 - u_1)$$

$$\left[F_N = \rho a (V_1 - u_1)^2 \sin\theta\right]$$

In x-direction-

$$F_x = F_N \sin\theta$$

$$F_x = \rho a (V_1 - u_1)^2 \sin^2\theta$$

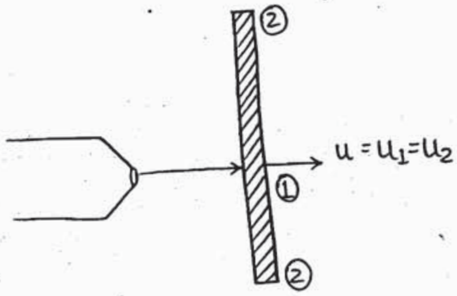
In Y-direction-

$$F_y = F_N \cos\theta$$

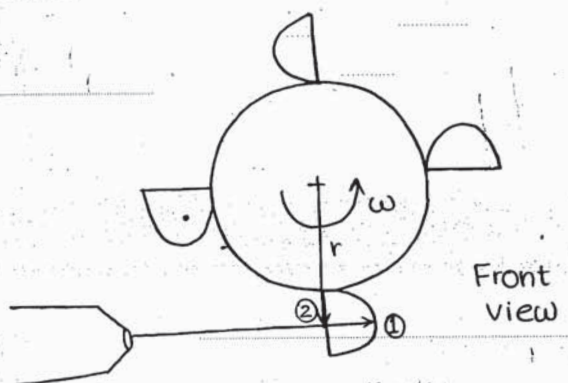
$$F_y = \rho a (V_1 - u_1)^2 \sin\theta\cos\theta$$

In General -

Single plate :-



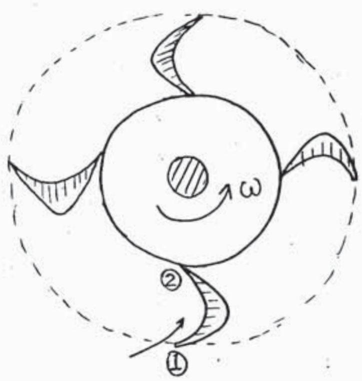

Pelton wheel :-




$[u = r\omega]$

$u_1 = u_2$

Inward Radial flow runner :-




Top view

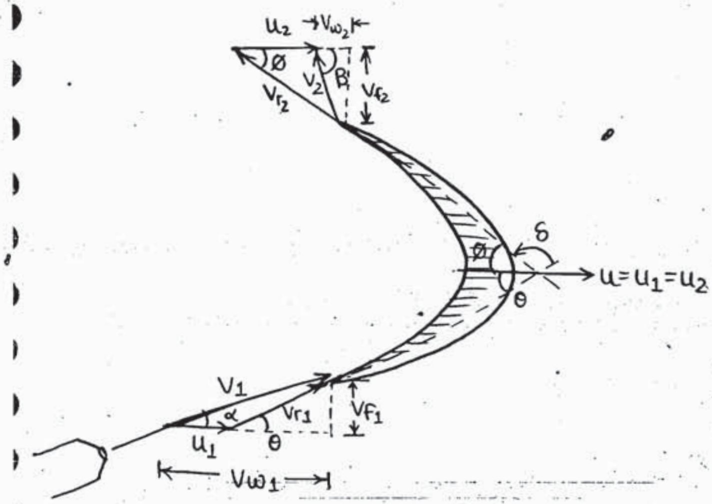
$R_1 > R_2$

$u_1 = \omega R_1 = \frac{\pi D_1 N}{60}$ $\quad$ $u_2 = \omega R_2 = \frac{\pi D_2 N}{60}$

$u_1 > u_2$

Single Surface :-






$\left.\begin{matrix}\theta \\ \phi\end{matrix}\right\}$ Vane Angle

$\alpha$ = the angle at which jet strikes the vane at inlet.

Note:- For Reaction turbine

$\alpha$ = Guide vane angle

$\delta$ = Angle of deflection

$V_w$ = velocity of whirl

$V_f$ = velocity of flow.

$$\dot{m} = \rho a V_{r_1}$$

In x-direction :-

$$F_x = \dot{m} V_1 \cos\alpha - (-\dot{m} V_2 \cos\beta)$$

$$= \dot{m} (\underbrace{V_1 \cos\alpha}_{V_{w_1}} + \underbrace{V_2 \cos\beta}_{V_{w_2}})$$

$$\left[ F_x = \dot{m} (V_{w_1} + V_{w_2}) \right]$$

In Y-direction :-

$$F_y = \dot{m} V_1 \sin\alpha - \dot{m} V_2 \sin\beta$$

$$= \dot{m} (V_1 \sin\alpha - V_2 \sin\beta)$$

$$\left[ F_y = \dot{m} (V_{f_1} - V_{f_2}) \right]$$

Work done per second by the water jet on the vane $= F_x \cdot u_1$

$$= \dot{m} (V_{w_1} + V_{w_2}) u_1$$

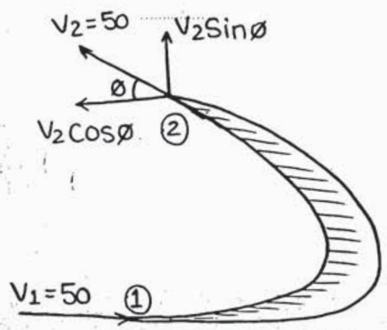
Work done per sec per unit weight of water striking per sec $= \dfrac{\overbrace{\dot{m}}^{\rho a V_{r_1}} (V_{w_1} + V_{w_2}) u_1}{\underbrace{\dot{m}}_{\rho a V_{r_1}} g}$ .

Efficiency ($\eta$):-

$$\eta = \frac{\text{Work done per sec}}{\text{input water power}}$$

$$= \frac{\overset{\rho a V_{r_1}}{\dot m}\,(V_{w_1}+V_{w_2})\,u_1}{\frac{1}{2}\,\underset{\rho a V_1}{\dot m_{Nozzle}}\,V_1^2}$$

T-1)
Pg-62)




Assume -
Smooth surface
$V_2 = V_1$

$$F_x = \dot m V_1 - (-\dot m V_2 \cos 25^\circ)$$

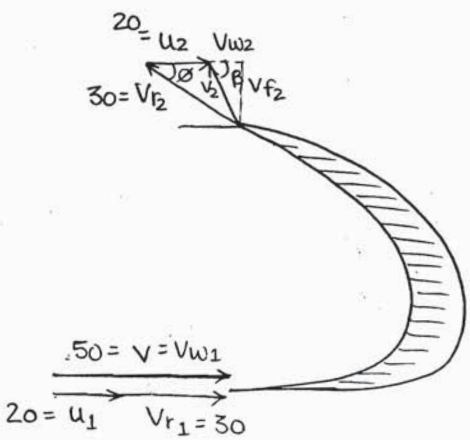
$$\frac{F_x}{\dot m g} = \frac{\dot m\,(50 + 50\cos 25^\circ)}{\dot m g} \quad N/N/s$$

$$= 9.72\ N/N/s$$

$$F_y = 0 - \dot m V_2 \sin 25^\circ$$

$$\frac{F_y}{\dot m g} = -\frac{\cancel{\dot m}\,(50 \sin 25^\circ)}{\cancel{\dot m}\,g}$$

$$F_y = -2.15\ N/N/s$$




assume - Smooth
Surface
$V_{r1} = V_{r2}$

$$F_x = \dot m\,(V_{w_1} + V_{w_2})$$

$$\frac{F_x}{\dot m g} = \frac{\cancel{\dot m}\,(50 + 7.19)}{\cancel{\dot m}\,g}$$

$$= 5.83\ N/N/s$$

$$\left\{ \begin{aligned} V_{w_2} &= V_{r_2}\cos\phi - u_2 \\ &= 30\cos 25 - 20 \\ &= 7.19\ m/s \end{aligned} \right.$$

$$F_y = \dot m\,(\cancelto{0}{V_{f_1}} - V_{f_2})$$

$$\frac{F_y}{\dot m g} = \frac{\cancel{\dot m}\,(-12.68)}{\cancel{\dot m}\,g}$$

$$\left\{ \begin{aligned} V_{f_2} &= V_{r_2}\sin\phi \\ &= 30 \sin 25^\circ \\ &= 12.68\ m/s \end{aligned} \right.$$

$$= -1.29\ N/N/s$$

Q-75)
Pg-61) $V_1 = 15\,m/s$ , frictionless vane $(V_{r_2} = V_{r_1})$

$u_1 = u_2 = 5\,m/s$

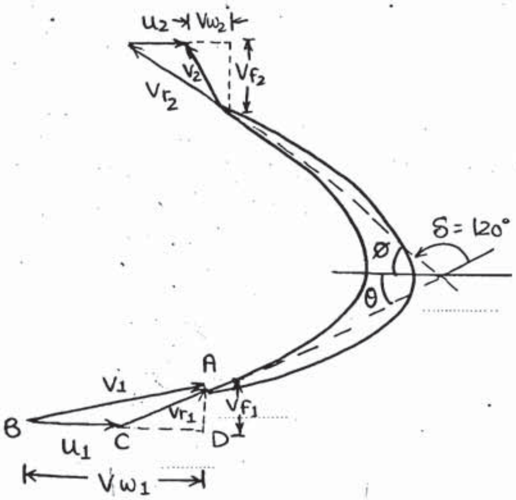
$\theta = \phi$

$\delta = 120^\circ$

determine -

$\alpha = ?$ , $V_2 = ?$

$\dfrac{\text{Work done per sec}}{\dot{m}g} = ?$




at inlet

- $\Delta ABC$
- Sine rule

$$\frac{V_1}{\sin 150^\circ} = \frac{u_1}{\sin(180-(150+\alpha))} \Rightarrow$$

$$\frac{15}{\sin 150} = \frac{5}{\sin(30-\alpha)}$$

$$[\alpha = 20.4^\circ] \text{ Ans}$$

$V_{w_1} = V_1 \cos\alpha$

$= 15 \cos 20.4^\circ$

$V_{w_1} = 14.06\,m/s$

$V_{f_1} = V_1 \sin\alpha$

$= 15 \sin 20.04$

$= 5.23\,m/s$

$$V_{r_1} = \frac{V_{f_1}}{\sin\theta}$$

$$= \frac{5.23}{\sin 30}$$

$$V_{r_1} = 10.46\,m/s$$

At Exit:-

$$V_{f_2} = V_{r_2} \sin\phi$$
$$= 10.46 \sin 30^\circ$$
$$= 5.23 \text{ m/s}$$

$$V_{w_2} = V_{r_2} \cos\phi - u_2$$
$$= 10.46 \cos 30^\circ - 5$$
$$= 4.06 \text{ m/s}$$

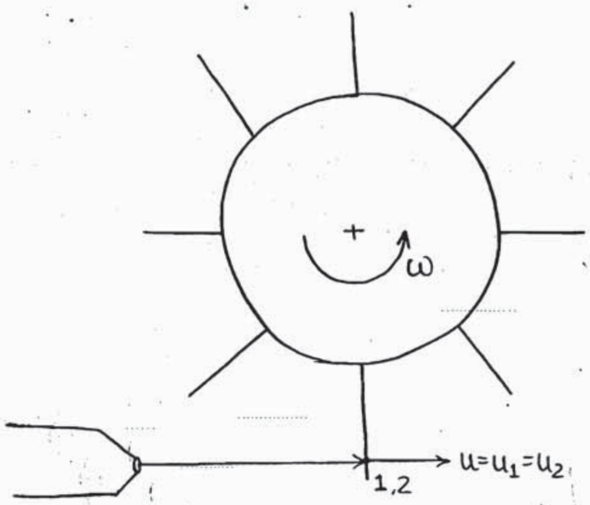
$$V_2 = \sqrt{V_{w_2}^2 + V_{f_2}^2}$$
$$= \sqrt{(4.06)^2 + (5.23)^2}$$
$$V_2 = 6.62 \text{ m/s}$$

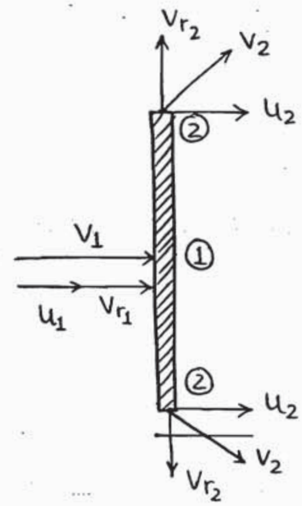
Work done per sec per unit weight of water striking per sec.

$$= \frac{\dot{m}(V_{w_1} + V_{w_2})u_1}{\dot{m}g}$$
$$= \frac{(14.06 + 4.06)}{9.81} \times 5$$
$$= 9.23 \text{m} \quad \underline{\text{Ans}}$$

## Impact of jet on a series of Flat plate or Runner






$$\dot m = \rho a V_1$$

$$F_x = \dot m V_1 - \left(\frac{\dot m}{2} u_2 + \frac{\dot m}{2} u_2\right)$$

$(u_2 = u_1)$

$$F_x = \dot m (V_1 - u_1)$$

Work done per sec or Runner power $= F_x u_1$

$$= \dot m (V_1 - u_1) u_1$$

Efficiency ($\eta$) :-

$$\eta = \frac{\text{Work done per sec}}{\text{input water power}}$$

$$= \frac{\overset{\rho a V_1}{\dot m} (V_1 - u_1) u_1}{\frac{1}{2} \underset{\rho a V_1}{\dot m_{Nozzle}} V_1^2}$$

$$\eta = \frac{2(V_1 - u_1) u_1}{V_1^2}$$

For Max $\eta$ -

$$\frac{d\eta}{du_1} = 0$$

$$\frac{d}{du_1}\left[\frac{2(V_1 - u_1) u_1}{V_1^2}\right] = 0$$