1. Determinacy & Indeterminacy

→ The structure in which all member forces and support reactions can not be found out using only the equations of static equilibrium is called "Indeterminate structure".

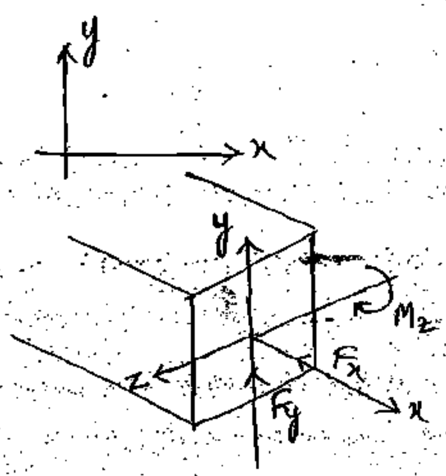
→ The equations of static equilibrium are;

$$\begin{array}{c|c} \Sigma F_x = 0 & \Sigma M_x = 0 \\ \Sigma F_y = 0 & \Sigma M_y = 0 \\ \Sigma F_z = 0 & \Sigma M_z = 0. \end{array}$$

→ However in 2D-structures, no. of equations of static equilibrium are;

$$\left.\begin{array}{l} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M_z = 0 \end{array}\right\}$$

→ when structure is in XY-plane



→ In structures we generally use Indeterminate structures.

→ In Indeterminate structures, BM developed is smaller, Hence the c/s requirement is less and Dead load of the structure reduces, also there are multiple paths of load transfer available. Hence failure of one member does not lead to the collapse of complete structure.

→ However in case of Indeterminate structures we need to make stronger supports and this will incure additional cost, Also the settlement of support (or) temp changes will give rise of additional stress.

> However Overall there is an economy in the structure.

In the Indeterminate structure there are two methods of Analysis.

1. Force method
2. Displacement method

→ In Force method, member forces (or) Support reactions are taken as unknown and Compatibility equation is written to find out the redundants.

→ No. of Compatibility equations required is equal to the degree of static indeterminacy of the structure

> In displacement method of Analysis, member and displacements are taken as unknowns and to find out these displacements we need to write equilibrium equations.

> No. of equilibrium equations required is equal to the degree of kinematic Indeterminacy of the Structure.

⇒ Degree of static Indeterminacy (D_s) :-

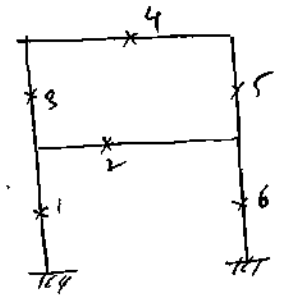
$$\boxed{D_s = \begin{bmatrix}\text{No. of unknown member forces} \\ \text{\& support reactions}\end{bmatrix} - \left[\text{No. of eqns of static equilibrium}\right]}$$

→ $\boxed{D_s = D_{se} + D_{si}}$

D_{se} = Degree of external Indeterminacy

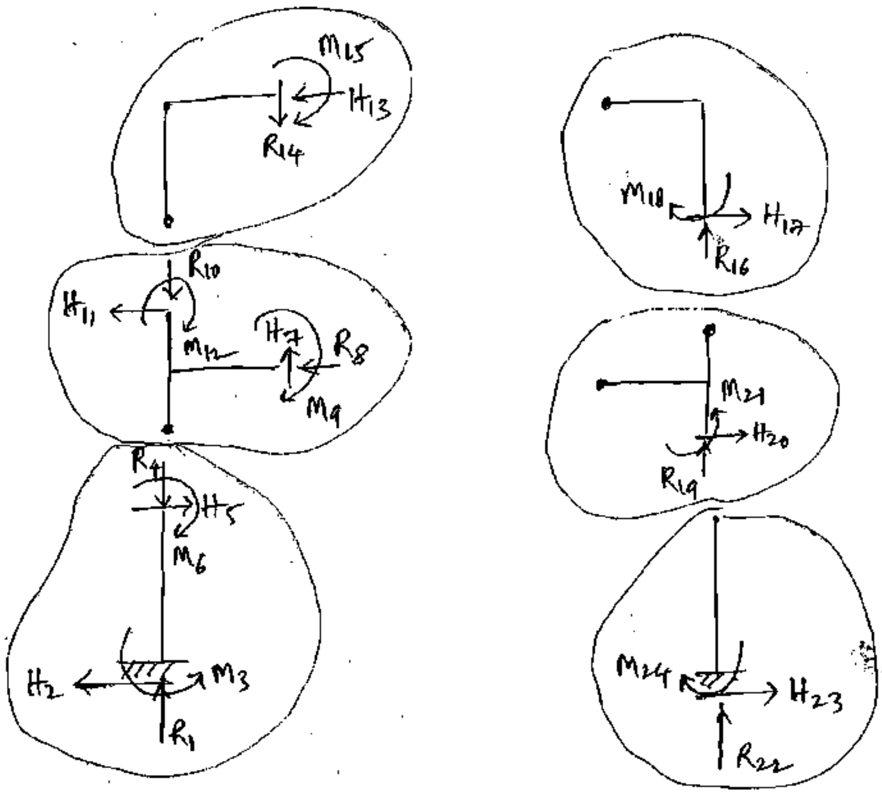
D_{si} = Degree of Internal indeterminacy

Ex:-



Here, 6 members

Sol:-



Total no. of member forces & support reactions $= 6\times3 + 6 = 24$

No. of equations of static equilibrium $= 6\times3 = 18$

$\therefore D_s = 24 - 18 = 6.$

→ D_{se} = Total no. of support reactions − no. of eq^ns of static equilibrium

$= 6-3 = 3$

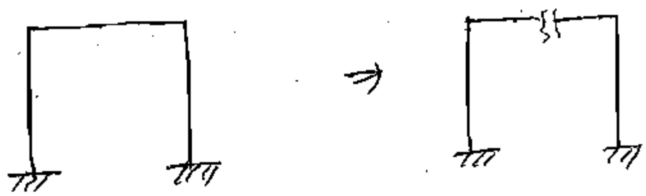
→ $D_{si} = 6-3 = 3$. (or) [No. of boxes × 3)

$= 1\times3$

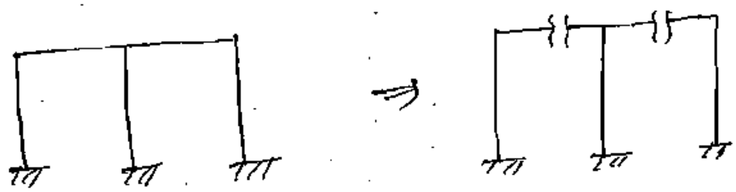
$= 3$

- If all the support reactions can be calculated only by using equations of static equilibrium, the structure is said to be "<u>Externally determinate</u>", otherwise "<u>Externally Indeterminate</u>".
- If by knowing all the support reactions we can findout all the member forces using equations of equilibrium, the structure said to be "<u>internally determinate</u>", otherwise "<u>Internally Indeterminate</u>".

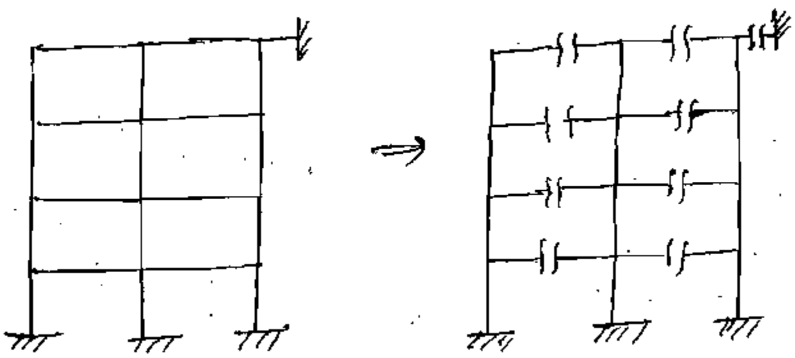
3) Static Indeterminacy of frames :

① 

$\boxed{D_s = 3}$ $\left(D_s = \underbrace{3}_{\text{Reactions}} \times \underbrace{1}_{\text{no. of cuts}}\right)$

② 

$D_s = 3 \times 2 = 6$

③ 

$D_s = 9 \times 3 = 27$

(or)

$D_s = D_{se} + D_{si}$

$= (3+3+3+3-3) + (\text{no. of boxes} \times 3)$

$= 9 + 6 \times 3 = 27$

→ In case of frames ; $\boxed{D_s = 3C - R'}$ → for 2D-structure

$= 6C - R'$ → for 3D-structure

where, C = No. of cut required to make Tree like structures

R' = No. of restraints required to make all joints rigid.

→ while making Tree like structure we must note that, one tree can have only one root, and there should be no closed loop branches. and none of the branches should fall off during cutting

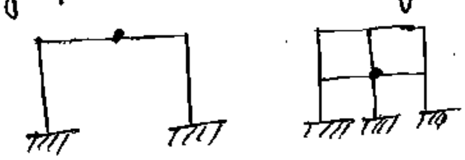
→ Restraining Supports :-

→ for no. of restraints required to make the support rigid equal to

$$= \begin{bmatrix} \text{No. of support reactions have} \\ \text{the support been fixed} \end{bmatrix} - \begin{bmatrix} \text{existing no. of support reactions} \end{bmatrix}$$

→ Restraining members (or) joints :-

→ In case of plane frame having internal hinge,

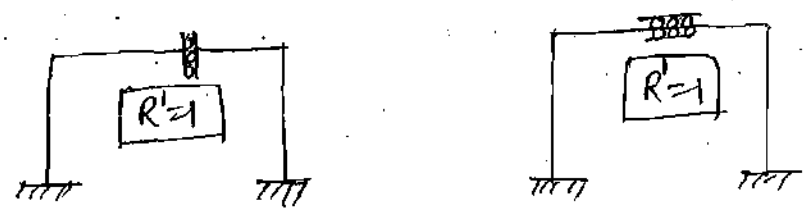


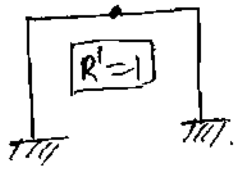
No. of restraint required = $m - 1$

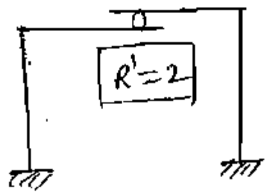
where, m = no. of members meeting at the joint

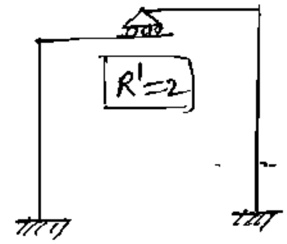
→ In case of space frame with internal hinge

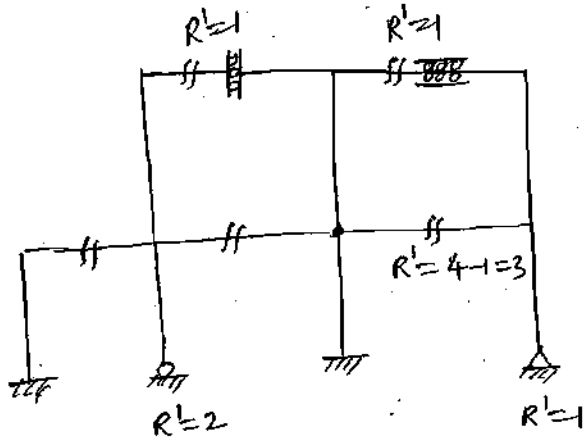
No. of restraint required = $3(m-1)$







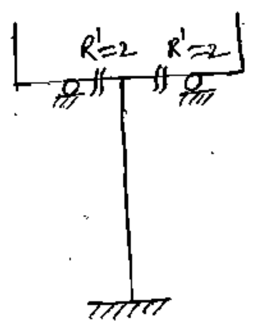




$D_s = 3C - R' = 3 \times 5 - (1+1+3+1+2) = 7$

$D_{se} = (3+1+3+2) - 3 = 6$

$D_{si} = D_s - D_{se} = 7 - 6 = 1$

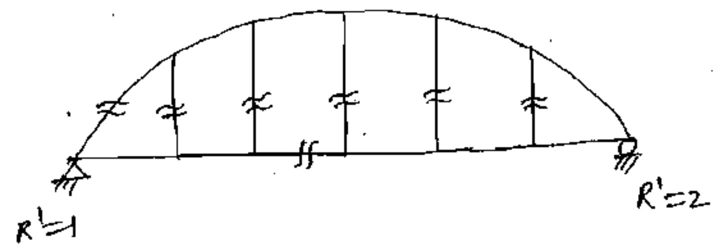


$D_s = 3C - R' = 3 \times 2 - (2+2) = 2$

$D_{se} = (3+1+1) - 3 = 2$

$D_{si} = 0$ [$\because$ because No boxes]

→



$D_{se} = 0$

$D_{si} = 6 \times 3 = 18$

$D_s = 18$

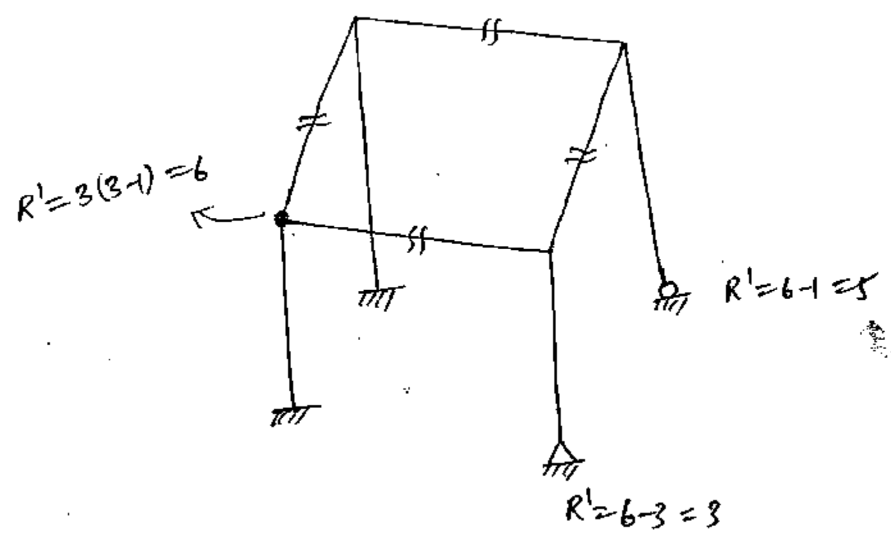
$D_s = 3C - R'$

$= 7 \times 3 - (1+2) = 18$

$D_{se} = 0$

$D_{si} = 18$

→



$D_s = 6C - R'$

$= 6(4) - (6+3+5)$

$= 10.$

→ $D_{se} = (6+6+3+1) - 6$ ← no. of static equilibrium eqns in space

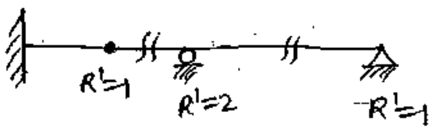
$= 10.$

3D

Actual procedure for D_{si} → $\therefore D_{si} = D_s - D_{se} = 10 - 10 = 0$ (in beams take $D_{si} = 0$, directly)

⇒ Static Indeterminacy of Beams :

› Static Indeterminacy in case of beams can be calculated using the same approach as for the frames.

→ 

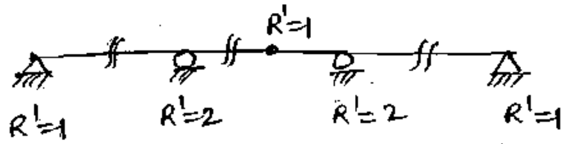
$C = 2,\ R' = 1+2+1 = 4$

$\boxed{D_s = 3C - R' = 3\times 2 - 4 = 2}$

Note:

› In case of beams internal indeterminacy can be taken as zero

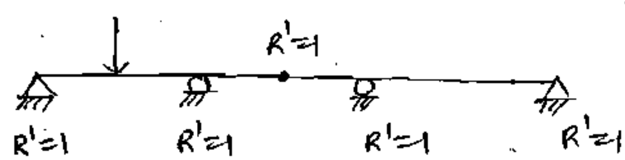
›



$C = 3,\ R' = 1+2+2+1+1 = 7$

$\boxed{D_s = 3C - R' = 3\times 3 - 7 = 2}$

→



(Here, no. of static equilibrium are 2 (i.e, H=0))

$\boxed{D_s = 2C - R'}$
$= 2\times 3 - 5$
$= 1$

te:

In case of beam having only vertical loading and all supports are being at same, the horizontal forces in the members and the support will not exist. hence D_s will be taken as $2C - R'$

i.e, $\boxed{D_s = 2C - R'}$